Automatic Control of Radiant PANEL HEATING

MINNEAPOLIS
Honeywell
CONTROL SYSTEMS

AUTOMATIC CONTROL
OF
RADIANT PANEL HEATING

A MANUAL OF THEORY AND APPLICATION

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Foreword

Radiant panel heating is at once one of the oldest and one of the newest methods of heating . . .

The Romans, throughout their empire, even as far afield as England, built central heating systems which were in all but one essential the forerunners of today's radiant panels.

The one basic element lacking in the ancient systems but now known to be essential is automatic control. It is in the interest of this phase of panel heating that this book has been written.

In keeping with the traditional policy pursued for six decades, of serving all branches of the heating industry with control equipment and technical data, Minneapolis-Honeywell offers this manual as an aid to all who are concerned with providing the utmost in comfort and efficiency with panel heating.

The inherent performance characteristics of a panel system give rise to control problems which are to a high degree peculiar to panel heating. In addition, the fact that the heating panel becomes in a real sense an integral part of the structure makes it especially important that the means, not to say the possibility, of achieving adequate control of a given installation be considered in the earliest stages of design. For this reason some space has been devoted to the relationship between design and control, from the standpoint of fundamental theory. To assure the best results, the design of the panel and the specification of controls should be closely coordinated.

In the first part of the manual, the theoretical basis of effective control for a panel heating installation is developed at some length, in terms of the thermal properties which also govern the design of the installation. The latter part includes:

- a set of three design graphs, introduced by an outline summary of the method of using the graphs for determining the required performance of the heating installation under control;

- a series of generalized control-system diagrams for various typical installations (such as single- and multiple-zone installations, with and without domestic hot water service, etc.), accompanied by brief descriptive comments.

These diagrams are intended to illustrate the general methods of control rather than to specify the actual instruments, which should be selected to fit the requirements of the individual installation.

In the preparation of this manual, invaluable assistance was obtained from Mr. F. W. Hutchinson, particularly in that portion of the manual dealing with the design of panel heating systems. Mr. Hutchinson's authoritative work with radiant heating problems is familiar to all who follow the technical literature of heating and air conditioning.

CONTENTS

Theory of Control of Panel Heating

The problem of providing suitable controls for a panel heating system is frequently considered to be a difficult one. The intent of this section is to demonstrate that the problem is *different* rather than difficult and that, assuming a correctly designed installation, once an understanding of the elementary principles of panel heating has been obtained, the application of these principles to problems of control will lead to a simple and direct solution. A comparative analysis of panel and convection heating systems, as to control requirement and inherent responsiveness to control, will provide a basis for a rational approach to the two major problems of control: (1) What type of control is required by the design of a given installation? And (2) what elements must be included in the control system to insure satisfactory response of the installation to control?

COMPARISON OF PANEL AND CONVECTION HEATING

The purpose of any heating system, in general space heating applications, is the same, whether the system is one of the familiar types depending chiefly upon convection for transfer of heat, or a panel system designed for a maximum of heat transfer by radiation. Since the primary difference, however, between the radiant and convective heating systems is in the manner in which the common purpose is carried out, it is desirable to examine that purpose in some detail before proceeding to a direct comparison of convective heating and radiant panel heating.

THE FUNCTION OF A HEATING SYSTEM

The function of any heating system, in residential, commercial, or industrial buildings, is summed up in the phrase "comfort heating." But what does *comfort* in this context mean?

In any environment having a temperature lower than 70° F the average lightly clothed adult at rest will lose body heat more rapidly than it is being produced. Under these conditions heat will leave storage in the body tissue, the body temperature will slowly decrease, and the occupant will become uncomfortably cold. The reverse is true when the temperature of the environment is much above 70° F. Body heat will be lost at a rate less than the rate of production until increased surface temperature and evaporation rate bring them into balance and the subject will feel uncomfortably warm.

The first of these conditions is of course the one that calls for artificial heating of an enclosure. In order to prevent discomfort it is necessary that the rates of production and dissipation of body heat be equalized. But the rate of production is fixed (with only minor exceptions) by the amount of work, either internal or external, which the subject is doing. Thus the only effective method of increasing the production of body heat would be for the subject to exercise, to do some form of external work which would necessitate marked physical effort. But he would then no longer be *at rest*. Obviously this method does not offer a satisfactory solution to the problem since the requirement of continuous physical exertion would merely substitute one kind of discomfort for another.

The other possibility of establishing a thermal balance is to prevent the rate of body heat loss from rising above a net value equal to that of body heat production. This can logically be accomplished by any of three methods: (1) Provide insulation (*i.e.*, more clothing) as the temperature of the environment declines, so as to keep the rate of heat transfer constant. (2) Supply heat to the environment as outdoor temperature declines, so as to prevent the decline of indoor temperature. (3) Supply heat directly to the occupant at a rate equal to the difference between the rates of heat loss to the environment and of heat production.

The first of these methods is the one used whenever a person goes outdoors in winter, but it is obviously unsatisfactory by present-day standards of indoor comfort.

The second and third methods both involve artificial heating. The second method is used with all non-radiant heating systems and with most floor panel systems. The third method by itself is not of practical importance since its use is limited to such special applications as high temperature spot heaters and similar local heating devices; but in combination with the second it is used as the basis of all ceiling or wall type panel heating installations (strictly considered), and the extent to which such systems approach the third method is an excellent criterion of their effectiveness. In *net* effect, however, as will be shown presently, all panel heating systems maintain thermal balance by the second method, maintaining the temperature of the environment at a level that permits the rate of body heat loss to remain equal to the rate of body heat production.

For practical purposes, then, convection and panel systems alike have the function of providing the additional heat necessary to maintain the temperature in an enclosure at the comfort level when outdoor conditions would otherwise cause uncomfortably coo

surroundings indoors. To understand the differences between convection and panel heating, however, it is necessary to examine more closely the temperature relations existing in a comfortable environment. In order to visualize the performance requirements of a heating system, consider the heat balance established by nature on the body of an average subject. Seated and at rest the thermal efficiency of the human body is approximately 20% and the requirements for internal work are such that the sum of the heat rejected directly plus that degraded as a result of internal friction amounts to 400 heat units (Btu) per hour. Of this heat quantity approximately 100 Btu are needed to provide the latent heat for the 1/10 lb per hour loss of moisture which occurs continuously through normal perspiration. Irrespective of temperature or of humidity, throughout the range of conditions in which a subject experiences reasonable comfort, the energy loss in the form of latent heat remains practically constant. Thus it follows that regardless of the method used to establish comfort within an enclosure this evaporative loss of body heat will always be approximately 100 Btu per hour.

The remaining 300 Btu per hour are dissipated by convection and radiation. In an environment with air at 70° F and all surfaces at 70° F the loss by convection from the clothed body amounts to approximately 140 Btu per hour, and the remaining 160 Btu per hour are dissipated by radiation to all surfaces which can be "seen" by the clothed body. Remembering that net heat transfer is always from the warmer to the cooler body, it follows that these rates of loss depend directly on the surface temperature of the subject in relation to air and surface temperatures of the environment. For an adult the average overall surface temperature (including clothed and exposed non-clothed surface such as hands and face) is close to 85° F. Assuming that the film coefficient of heat transfer by convection (or *unit convection conductance,* in Btu/hour/square foot/degree F temperature difference) and the equivalent film coefficient of radiant transfer do not appreciably vary with small changes in temperature difference and noting that the effective areas of the human body for convective transfer and for radiant transfer, though differing from one another, remain constant, it follows that the rates of body heat loss by convection and by radiation will, for a given change in the temperature of both air and environment surface, change in the ratio of 140 to 160; thus a change of 0.88° F in the temperature of all surfaces of the enclosure would be equal in its effect on radiant loss to a 1.0° F change in air temperature in *its* effect on convective loss.

Although the foregoing relationship does not hold exactly (for the reason that the temperature of the clothed body surface is more responsive to changes in air temperature than to changes in surface temperature), there is a substantial mass of experimental data to show that for all practical purposes the influences of air and of surface temperature are very nearly the same. On this basis a 1° F rise in the temperature of the enclosure surface would reduce the radiant loss of body heat by an amount substantially equal to the increased loss occasioned by a 1° F reduction in air temperature. This relationship is of the greatest practical importance and is of significance in obtaining controls that will provide optimum comfort from any heating system, regardless of its type. Thus if the overall surface temperature of the enclosure decreases in very cold weather (as it does with conventional types of convection heating systems because of the marked reduction of the inside surface temperature of exterior walls and floors), the radiant loss of body heat from the occupant will be increased, and the inside air temperature must be raised sufficiently to compensate for the "cold wall effect". Conversely, if the inside air temperature falls, an increase in the surface temperature of the enclosure is necessary for maintaining optimum comfort, so as to reduce radiant loss of body heat as much as the convective loss is increased.

Because of its importance in design and in establishing control criteria the air-temperature to surface-temperature relationship just discussed is usually expressed mathematically in the so-called Comfort Equation,

$$t_a = 140 - t_m \qquad \text{Equation (1)}$$

where t_a is the air temperature at breathing level within the enclosure, t_m is the average surface temperature of the enclosure, and the factor 140 is based on the observation that the average adult is comfortable when $(t_a + t_m)/2 = 70$. Technically, the term t_m represents a mean radiant temperature measured with respect to the occupant and varying with changes in his position in the room—*i.e.,* with changes in the distance between his body and each point on the surrounding surfaces. For all practical purposes, however, t_m can be considered as the arithmetical weighted average of the room surface temperatures (the sum of the products of temperature and area of the several surfaces, divided by the total surface area). The Comfort Equation as given is applicable to any room in which the air movement is not unusually great. For special cases involving high air velocities across the occupants—such as may occur in industrial installations—the equation loses its validity because of the increased film coefficient and the consequent added importance of convection losses. In general, however, the Comfort Equation expresses with suffi-

cient accuracy the performance requirements of any automatic heating system; namely, the maintenance of such a relationship between air temperature and surface temperatures in the enclosure that their combined effect on the rate of body heat loss will always be equivalent to the effects of air and surface temperatures of 70° F.

CONTROL REQUIREMENTS OF THE TWO SYSTEMS

In terms of the Comfort Equation the primary difference as well as the similarities between panel heating and other heating methods can be readily brought out. Differences in the manner in which the heating system determines air and surface temperatures necessarily entail differences in design procedure for the two types of installations. What is more important for the present purpose, they also entail differences in the type of control required to ensure that the heating system will supply heat at the rate necessary to the maintenance of the desired temperature relations under varying conditions of heat loss from the enclosure to the outdoors. Since an understanding of these differences is of primary importance to the effective application of controls to a panel heating installation, it is desirable to consider separately the performance characteristics of convection systems and of panel systems before attempting to sum up the differences as to the primary objects of control.

Temperature Relations in Convection Heating

Since all heating systems of the convection type (radiators, convectors, warm air furnaces, etc.) accomplish heat transfer to the environment primarily by convection, it follows that only the air temperature is directly determined by the heating system. The inside surface temperature varies as a function of the inside air temperature, and also as a function of the outside air temperature (as well as wind velocity, "sun load," etc.). The exact relation depends upon the thermal characteristics of the structure. For convenience, however, the designer of any type of convective heating system would select some value of the inside air temperature (usually 70° F), calculate by means of the thermal characteristics of the structure the heat input to the room needed to maintain the inside air temperature at the selected value under assumed full load conditions (design temperature), and then determine the expected average surface temperature under the same load and input conditions. If this average surface temperature were appreciably lower than 70° F he would then use the comfort equation to determine how much the air temperature should be raised at design load to permit realizing conditions of optimum comfort. Thus although the designer's primary interest is in the room air temperature he must nevertheless adjust this temperature in such a manner as the thermal characteristics of the structure (with respect to inside surface temperature) may dictate. Since the air temperature and the average surface temperature are not independent variables, an accurate design must provide compensation of the design air temperature for "cold wall effect." But the inside surface temperature varies continuously as a function of the outside air temperature; so it is evident that in order to maintain optimum comfort within the enclosure at all times it would be necessary to increase the inside air temperature continuously as the outside air temperature decreased. Actually, this requirement rarely occurs in practice since, in structures which are adequately insulated and do not have unusually large areas of single glass windows, the overall decline in temperature of inside surfaces is not appreciable. Ideally, however, optimum comfort would require that *any* convection type heating system be controlled so that the inside air temperature is held at 70° F when the outside air temperature is 70° F, and raised slightly but continuously as the outside temperature decreases.

Temperature Relations in Combination Systems

A special type of panel system of increasing importance may be considered at this point, particularly since the control requirements for such an installation are very much simpler than for other types, and serve to clarify the contrast between convection and panel systems. In large industrial working spaces where the ratio of occupants to cubic contents is low and where the workers retain relatively fixed positions it is both practical and highly profitable (in terms of comfort and of heating costs) to use localized overhead or floor panels of capacity sufficient to maintain the warmth of the worker at any station without consideration of the temperature level in the room. In such installations it is customary to maintain the air temperature in the room at some relatively low fixed value such as 55° F. The opportunity for economy increases as the inside temperature is lowered, but 55° F seems to be the minimum acceptable value, for at lower temperatures the objects which must be handled by the workers are so cold that working efficiencies may be lowered. If, then, some type of conventional heating system is used and the room air temperature thermostatically held at 55° F, the problem of design resolves itself into one of providing individual station panels of sufficient area and temperature to make up to the workman the excess loss of body heat to the room air and to surrounding surfaces other than the heating panel.

The great advantage afforded by installations of this kind arises from the fact that all of the energy dissipated by the panel but not received directly by the occupants appears—unit for unit—as a reduction in the energy requirements of the supplementary convective heating system. Hence there are no losses from the panel and its net effect is to provide a heating "dividend" equal in amount to the energy that would be required to heat the structure by convection through the temperature range from 55° F to 70° F. If the supplementary convection requirements were less than the convection dissipation from the local panels the saving would, of course, decrease since it would then be impossible to prevent the room air temperature from exceeding 55° F, but in any work space large enough to permit consideration of local heating procedures it will always be found that the entire "loss" from the panel (including that part of the radiant dissipation which is not received directly by the occupant) will be insufficient to make up the requirements for supplementary heating. Thus the magnitude of the available saving is great, varying (except for the times during the heating season when the outside temperature is above the critical value for inside air temperature of 55° F) in the ratio of (70-55) to (70 minus normal outside temperature). For any given industrial application of this kind there will always be some critical value of the outside air temperature corresponding to which the energy dissipated by the panels will exactly equal the heat required to maintain an inside air temperature of 55° F. For all outside temperatures greater than this critical value the room air temperature will rise, the panel surface temperature will have to be reduced, and the saving over conventional heating will decrease.

The simplicity of control requirements where such "spot" panels are used is evident in the temperature relations indicated. The room air temperature is thermostatically held at the fixed value t_r. Since the panel is not required to supply the heat requirements of the structure but simply to make up to the occupant directly the excess loss of body heat to the remainder of the environment, it follows that the surface temperature of the panel should likewise be held constant at its design value (usually 120° F for overhead panels) irrespective of outside conditions, except when the critical outside temperature is exceeded and the inside air temperature thereupon starts to rise. When this occurs the panel temperature must, if optimum comfort is to be maintained, be continuously reduced until (for room air at 70° F) the panel surface temperature becomes 70° F. Thus it is evident that the controls must reduce the surface temperature by 50° F, during the period that the room air temperature is rising through the temperature range 70-t_r. Strictly, the variation is not a straight line proportion, but for all practical purposes in industrial comfort work, adequate control will be attained if the panel surface temperature is lowered 10% of (120°-70°) as the air temperature rises 10% of (70°-t_r).

What should be particularly observed is that, through the greater part of the range of outdoor temperature, *i.e.*, of heating load, the room air temperature and the surface temperature of the local panel are independently controlled *at fixed values*, since the air temperature is primarily determined not by the heating panel but by a separate convection heating system. Only in the critical range of load does the panel determine the room air temperature, so that the temperature of the panel must be reduced with a rise in room air temperature or a decrease in load.

Temperature Relations in Panel Heating

Turning now to the subject with which this manual is directly concerned, panel heating in the sense of comfort space warming, the problem of control as well as the problem of design is considerably more complex than it was found to be for local "spot" panels in industrial applications. In general space heating applications—such as are found in residences, office buildings, hospitals, and public buildings—the design problem must be broadened to take account of the thermal and ventilation characteristics of the structure, since no auxiliary air heating system is available and the panels must alone establish conditions of optimum comfort uniformly throughout the occupied space.

In effect, all types of panel heating systems establish a balance between body heat production and body heat dissipation by the second of the two methods outlined earlier—that of supplying heat to the environment. Because of the relatively large amount of heat that leaves a panel by convection (28% for a ceiling panel, 41% for a wall panel, and 48% for a floor panel) it is usually found that the equilibrium value of the inside air temperature is no lower than 63° F, and the average inside surface temperature, to satisfy the Comfort Equation, may not exceed approximately 77° F. But the clothed body surface temperature is approximately 85° F. Therefore, even though wall and ceiling panels operate, for loads greater than approximately 30% of design maximum, at surface temperatures sufficiently above 85° F (usually 100° and 120° maximum respectively) so that the actual net transfer between the occupant and the *panel* is *to* the occupant—*i.e.*, irradiation of the occupant by the panel—nevertheless the *net* radiant transfer, as well as the convective transfer,

between the occupant and the total environment is to the environment. What is thus true of the overall result is likewise true in an exact sense for floor type panel heating systems, since comfort limitations require that the panel be of sufficient area to permit operation under full load with a surface temperature not greater than 85° F (excepting margins and other areas seldom walked on); hence the net radiant transfer between an occupant and a floor panel is always away from the occupant and for such a system there is no net transfer to the occupant from any element of the environment.

The fact that wall and ceiling panels provide some direct irradiation—*i.e.,* radiant heat transfer *to* the occupant—constitutes the most significant difference, from the point of view of theory, between wall or ceiling panels and floor panels. It suggests the interesting possibility that, in the future, methods may be developed for reducing the ratio of convective to radiant dissipation of energy from a wall or ceiling panel so that the system will operate essentially by the third method—direct make-up of heat loss—and hence permit control of an environment solely in terms of the required energy make-up to the occupants. Of the methods that have been suggested, however, none at present holds much promise. For one thing, not only must the portion of the energy dissipated from the panel by radiation as compared with the convective fraction be increased, but the fraction of the total radiated energy actually received by the occupant must be significantly increased from the present value of approximately ½ of 1% (from a ceiling panel). The proposal to insulate the panel from the air by a material transparent to infra-red depends for its practical application on the development of a suitable material whose cost would not be prohibitive. In addition, the reduction in *total* heat transfer would require increasing the rating of the panel by increasing the surface temperature (already limited by the difficulty of providing uniform comfort in all parts of the room) or the area of the panel (already approaching the total area of the ceiling in localities where the design temperature is lower than −10° F). Another proposal, the use of reflective materials on the unheated surfaces to prevent or delay the absorption of radiant energy emanating from the panel, is shown by mathematical analysis and experiment to be of no practical value, for the designer or the control engineer*. On the contrary, the designer can accurately treat all unheated surfaces of an average room as though they were "black bodies"—*i.e.,* perfect emitters and absorbers of radiant energy.

*Hutchinson, "Influence of Gaseous Radiation in Panel Heating." ASHVE Journal Section, *Heating, Piping and Air Conditioning,* November 1946.

This possibility also simplifies the problem of estimating performance and providing controls that will give that performance. Moreover, the purpose of this manual is to facilitate the selection and application of controls for panel heating installations of the types now available.

With existing methods, then, the heating panel dissipates a significant fraction of its energy output by convection, in addition to the energy dissipated by radiation. Consequently the room air temperature is primarily determined by the panel directly, and at the same time the average surface temperature of the room is determined in part by the air temperature, in part by direct radiant transfer of heat from the panel to the unheated surfaces, and in part by the temperature of the panel as a factor in the overall average, since the area of the panel is a considerable fraction of the total surface area. It follows, therefore, that a given change in panel temperature will in general have more influence on overall surface temperature than on air temperature, although the difference varies both with the location of the panel and with the relative magnitude of the convective heating load, which in turn depends largely on the ventilation factor. One way of visualizing the significance of this relationship is to consider the result of ignoring it in an extreme case—say a room with a high ventilation rate, although well insulated, and heated by means of a ceiling panel. Remembering that the proportion of convective to radiant heating with such a panel is less than 1:2, consider what would happen if it were attempted to maintain a constant air temperature of 70° F as the outside air temperature declined to the design value of, say, −10° F. As the total input to the panel was increased sufficiently to meet the increased convective heating load, the radiant input to the room would be increased in excess of the requirement for maintaining an average surface temperature of 70° F, and marked overheating would be experienced. Conversely, if the panel temperature were controlled so as to maintain the average surface temperature at a constant value of 70° F, the increase of convective input to the room would be insufficient to prevent the room air temperature from falling far below the comfort level. Whereas in convection heating under thermostatic control the average surface temperature tends to fall more and more below the room air temperature as the outside temperature falls toward the design value, in panel heating the overall average surface temperature tends to rise as the heat input is increased to meet an increase in heating load. Consequently, to satisfy the Comfort Equation, the room air temperature should be allowed to fall just enough to compensate for the increase in surface

temperature. The necessary depression of air temperature below 70° F will often be found to be from 5° to 7° F.

The Primary Objects of Control

The foregoing discussion may be summarized as follows: Optimum comfort may be realized in an enclosure with general space heating, irrespective of the type of heating system used, only if the room air temperature is continuously varied as a function of outside air temperature, and in a direction opposite to the trend of average inside surface temperature. For convective heating systems the departure of air temperature from 70° F as load increases should be upward, but, except in very unusual structures, it is so small as to be of no practical significance. Because of its relative unimportance, most designers of convective type systems give little attention to the air temperature variation, but design for a fixed value such as 70° F, and specify controls intended to maintain that temperature in the enclosure for all loads. For panel heated structures the situation reverses. The departure of optimum inside air temperature from 70° F as the load increases is downward and is frequently of a magnitude which requires that means of control be provided to permit continuous variation of this temperature as a function of the load—usually assumed to vary as a function of the outside air temperature.

From the point of view of the control engineer this contrast between convection and panel heating systems is of primary importance. In convection heating, room air temperature is usually an adequate criterion of control. Panel heating, however, requires that the correct relationship be maintained between the panel surface temperature and the room air temperature, for a given heating load—something which is not always easy to do; and hence the air temperature alone can never be regarded as an adequate criterion of control.

The inadequacy of air temperature as sole criterion of control was strikingly illustrated by a recent practical problem: A five-room residence was heated by means of hot water coils embedded in the ceiling. The occupants found that, at an outside temperature of 30° F, optimum comfort was attained with a living room air temperature of 67° F. The thermostat was set at this temperature and controls provided which made it possible to maintain a living room air temperature within a fraction of a degree of 67° F, yet the occupants found that comfortable conditions very rarely existed in the house even during extended periods of time in which the outside air temperature remained practically constant at 30° F. The room was by turns uncomfortably warm and uncomfortably cool, *even though there had been no significant change in either the outside or the inside air temperature*. As will be shown later, the thermal characteristics of the structure and of the system were perfectly adapted to "hunting" of the panel temperature. The difficulty was finally corrected by changing the method of control. The circumstances are mentioned here to emphasize the fact that with panel heating the maintenance of constant air temperature—even at constant heating load—is not in itself assurance that the control system will be satisfactory. It is the relationship between air and surface temperatures that must ultimately be controlled, even though it may be done indirectly.

Later it will be shown that air temperature can be used *indirectly* as a criterion of control, but only by a designer who provides a control system which will establish the special relationships that make this possible. Add to this the necessity of lowering the air temperature gradually as the load increases, and it should be obvious why, until the simple principles of the panel system are understood, there is always very grave danger of mis-using thermostatic control and thereby introducing conspicuously unsatisfactory performance of the heating system.

INHERENT CONTROLLABILITY OF THE TWO SYSTEMS

Besides the fact that with a radiant system the object of control is the whole environment rather than the air temperature alone, a panel heating system poses a second problem arising out of the differences between panel and convection systems. The second problem is that of response to the controls.

During the transient period associated with solar gain, with changing outside air temperature, or with marked changes in the internal heat load, comfort can be attained only by achieving rapid response of the heating system to variations in load. A panel is usually installed as part of the structure, and is therefore likely to have large mass and hence to act as a reservoir of heat. Because of its greater thermal capacity a panel system of the usual type is inherently sluggish in its response to load changes and the probability of hunting is great unless special consideration is given to an anticipatory feature in the control system. There are, of course, many heating systems of conventional type in which the problem of slow response of the boiler or furnace may require careful control design, but the panel system is unique in that the thermal inertia of the system is concentrated at the most undesirable location; that is, at the heat-dissipating element, the panel, located *in the occupied space*. Ineffective or questionably satisfactory control in the boiler or furnace room may

escape detection for a long time, but unsatisfactory control of a heating panel located directly over the head or under the feet of the occupant is more than likely to bring an immediate demand for correction.

RATIONAL SOLUTION OF THE CONTROL POINT PROBLEM

The selection of control instruments in the proper combination to provide satisfactory performance from a given panel heating installation depends finally upon analysis of the characteristics of the system and of the structure that influence the response to changing load conditions. But the first and fundamental control problem is that of *control point characteristics:* What temperature relationship must be maintained for a satisfactory approximation to optimum comfort?

In the preceding section attention has been called to the fact that the air temperature in a panel heated space is less—for optimum comfort—than in a space heated by convection. In order to provide adequate control the first thing that must be determined is, "How much less?" But the answer to this question can only be obtained from an understanding and application of the fundamental design procedure. Further, the air temperature reduction decreases as the outside air temperature increases and if adequate control is to be maintained it is essential that the path along which the inside air temperature moves be fixed from a heat balance analysis in terms of the path of outside air temperature. Should the inside air temperature be allowed to rise as a straight line function of outside air temperature? An adequate answer to this question can only be obtained from critical examination of the heat balance equations leading to a determination of the relationship corresponding to optimum comfort.

It will be found that, when the required reduction of room air temperature at design load is small, control in terms of fixed air temperature will prove acceptable. But first it is obviously necessary to determine accurately the magnitude of the depression of air temperature called for by the Comfort Equation.

A detailed analysis of design is not within the scope of a manual of control methods but, as the foregoing indicates, control is itself inseparably related to the factors which influence design; indeed, the possibility of achieving satisfactory control is determined by the design. Therefore a clear understanding of the control procedure may best be obtained through a review of the primary heat balance equations and of the rational design procedure derived from them.

INADEQUACY OF CONVENTIONAL DESIGN PROCEDURE

One might suppose that the design of a panel heating system would logically be carried out by the reverse of the method described earlier for convection systems. That is, the designer would arbitrarily select, not a value for the inside air temperature, but an average inside surface temperature, and provide a heated panel of sufficient temperature to raise the average surface temperature of the enclosure to the selected value. For optimum comfort the air temperature within the room should then be maintained at a value lower than 70° F by as many degrees as the average surface temperature is above 70° F. But how can this be done? Once the panel area and temperature have been fixed the room air temperature is no longer an independent variable. By carrying out a heat balance on the room the designer could calculate the air temperature at which equilibrium would be established when the average surface temperature was maintained at any selected value. But if these values did not then satisfy the Comfort Equation (and they would satisfy it only by chance) he would be unable to do anything about it except to abandon his initial assumption as to surface temperature and make a second trial. For any given structure under any particular set of operating conditions (as determined by outside air temperature and inside ventilation rate, or infiltration rate) there is *one and only one* combination of surface temperature and air temperature which can satisfy the heat balance equations and also the comfort equation.

On the other hand, any design procedure which starts with an assumed value of the inside air temperature is inherently wrong and in many practical cases, particularly for structures having a high ventilation rate, the magnitude of the resultant error may be serious. Unlike convective heating systems, as has been shown, panel heating systems frequently operate with average surface temperatures sufficiently above 70° F to introduce substantial operating difficulties if the inside air temperature is not changed from 70° F to a lower value that will satisfy the Comfort Equation. In sharp contrast with convective systems, a panel system accurately controlled will usually operate with an air temperature less than 70° F and the magnitude of the reduction will increase as the outside air temperature decreases. Thus for optimum comfort a panel heated room will have its *minimum* air temperature when under maximum load, in contrast with a convection heated room for which the *maximum* air temperature should correspond to maximum load. The minimum air temperature is precisely what must be determined, particularly for

purposes of control; and using an assumed value in the calculations will again result in a tedious series of trials.

Thus in designing a panel heating system it is incorrect to select either a design value of the inside air temperature or of the inside surface temperature; an accurate design must be carried out by heat balances in which the panel area (or temperature) and the inside air temperature are unknowns to be evaluated as a result of the design procedure.

RATIONAL METHODS OF PANEL DESIGN

A number of different methods have been developed for analysis and design of panel heating systems. European writers have usually made use of a semi-empirical method whereas the trends in the United States have been, on the one hand, toward fully empirical "rule-of-thumb" methods and, on the other, toward rational analytical procedures. Eventually, as experience with panel systems for all types of structures and for all kinds of operating conditions is acquired and analyzed, it will undoubtedly be possible to develop a simplified empirical (*i.e.*, "practical") design procedure that will be short, reasonably accurate, and sufficiently conservative for general use. At the present stage of development of the art, however, methods based on experience and observation are not only of dubious value but actually dangerous, since the variety of experience on which they are based is not sufficient to assure their applicability to all ordinary design problems. For the present purpose, therefore, a rational, mathematical analysis of design and control procedures is preferable to an empirical approach to the problem.

Available rational methods are of different forms, but all derive from the concept of setting up heat balance equations on various parts of the enclosure for which the design is to be carried out, and then solving these equations simultaneously to determine the requisite panel area, or surface temperature (one or the other of these independent variables must be arbitrarily fixed, or fixed in terms of some non-thermal limitation). A rational method of this kind is described in detail in a series of papers which have appeared in the *Transactions* of the American Society of Heating and Ventilating Engineers over the period from 1941 through 1946. The most serious disadvantage of such a procedure is the tediousness of computations resulting from the necessity of evaluating the shape factor of each different type of room surface with respect to each other type of surface making up the enclosure. Since an average room is likely to have five different types of surface (glass, exposed wall, interior partition, floor, unheated ceiling) and since two additional equations must be written (one for ventilation air passing through the room and another to express the comfort relationship), it is evident that the routine computation involved in the simultaneous solution of seven equations is, in itself, no small task.

Fortunately, a simplification of the rational procedure can be achieved which vastly reduces the required amount of work without causing any significant loss of either accuracy or generality. A clue to this simplification is found in the fact, already mentioned, that the radiant behavior of any average size enclosure is practically the same as that of a "black" walled enclosure irrespective of the emissivities (or reflectivities) of the actual surfacing materials. In somewhat the same way—though the analogy is not an exact one—the respective influences of the different inside surface temperatures of the five types of unheated enclosure surfaces integrate in such a way that the overall effect does not differ materially from that which would occur in a room with all surfaces uniform, homogeneous, and at the same inside surface temperature. This comparison is not exact since in actuality there are not merely as many different inside surface temperatures as there are different types of surfaces, but there are an infinite number of surface temperatures, since temperature is a point function and will vary with respect to position on a surface of given type because of changing shape factor of the point in question with respect to the heating panel.

For all practical purposes, however, the thermal characteristics of any actual enclosure can be accurately idealized (and simplified) to an equivalent enclosure of exactly the same size and shape, but made up of only two different surfaces: the heated panel which is assumed at the uniform surface temperature, t_p, and all other parts of the enclosure (the total of unheated inside surface) which are likewise assumed to be at some uniform and (for fixed panel temperature) constant surface temperature, t_e. A proof of the accuracy and applicability of this concept is beyond the scope of this manual, but the interested reader will find in the literature a detailed discussion of it, with examples which permit comparison between the exact seven equation procedure and the procedure based on the concept of an idealized equivalent room.*

Assuming, then, that the use of an "equivalent" room, simplified in construction but having the same thermal characteristics as the actual complex room,

*Raber and Hutchinson. "Load Calculation Procedure for Electric Panel Space Heating." *Transactions of the American Institute of Electrical Engineers*, 1944.

Hutchinson, "A Single-Equation Design Procedure for Radiant Panel Systems." *Transactions of the American Society of Heating and Ventilating Engineers*, 1946.

can be justified, it is then possible to apply the rational method of analysis to this simplified room by writing only three equations. Thus one equation can be established by setting up a heat balance on the unheated room surface, and a second equation by writing a heat balance on the outside air which passes through the room either as a result of mechanical ventilation, or as infiltration and exfiltration through cracks around doors and windows. These two equations define fully the heat transfer characteristics of the room, but before using them in design or in investigation of control characteristics of the structure it is first necessary to provide a limitation requiring that conditions within the room satisfy the comfort requirements of the occupant as well as the thermal characteristics of the structure. The third required equation is therefore the Comfort Equation (equation 1) and, like the first two, this equation can be regarded as a heat balance; a heat balance written on the occupant.

Of these three equations, note particularly that the first two fully solve the problem of determining the relationships among the various surface temperatures and the inside air temperature of the enclosure. The third equation does not add new information, but it assists the designer by getting rid of the infinite number of possible equilibrium conditions which could thermally exist within the structure, but for which comfort would not be experienced. The third or comfort equation provides a limitation on the solution and, as will be shown, limits the resultant solution to the one set of air and surface temperatures (for given panel area) that will simultaneously satisfy the thermal characteristics of the structure and the comfort requirements of the occupants. It was for this reason that the statement was previously made that the designer does not have freedom to select in advance the inside air temperature because there is only one single value of this air temperature that, at any particular load, will permit realization of optimum comfort in that structure.

DERIVATION OF EQUATIONS FOR THE RATIONAL DESIGN PROCEDURE

Before setting up the three basic equations it is first necessary to define two fundamental terms that continually find use in the design or analysis of any panel heating system. These are the equivalent overall coefficient of heat transfer and the ventilation factor.

The Equivalent Overall Coefficient of Heat Transfer, U_e

Consider a room having the five different types of unheated surfaces which were discussed in the previous section and let the overall coefficients of heat transfer of these five surfaces (glass, floor, wall, partition, ceiling) be represented by U_g, U_f, U_w, U_i, U_c where the subscript i refers to an inside wall, and each area is considered as only that part of the respective surface which is not covered by heating panels. By the usual methods of calculating heat loads for convection heating the total transmission losses from such a room (if convection heated) would be,

$$q=(U_gA_g+U_fA_f+U_wA_w+U_iA_i+U_cA_c)\,(t_a-t_o) \tag{2}$$

where the subscripts a and o represent inside air and outside air temperatures. Again considering convection heating, the same thermal losses would occur from a homogeneous structure having a uniform overall coefficient of heat transfer, U_e, such that

$$U_e=\frac{(U_gA_g+U_fA_f+U_wA_w+U_iA_i+U_cA_c)}{A_t} \tag{3}$$

where A_t is the sum of the five areas through which transmission losses are occurring. Hence equation 2 can be simplified, for convection heating, to the form,

$$q=U_eA_t\,(t_a-t_o) \tag{4}$$

Equation 4 is not generally used in design of convection type systems since evaluation of the U_e term requires only slightly less computation than direct solution for transmission losses from equation 2. Neither is equation 4 used for computing transmission losses in a panel heated structure since, for this case, direct radiant transfer to unheated surfaces from the heated panel will alter the value of the equivalent inside film coefficient for radiant transfer and will thereby change the value of U_e.

Equation 3, however, is of inestimable aid to the panel designer since it provides a definition of an "equivalent" room that can serve as the starting point in panel analysis. The defined term, U_e, does not itself apply to a panel heated room, but from it can be obtained an "equivalent conductance" which is of great importance in panel calculations. The equivalent conductance, C_e, is the rate of heat transfer, expressed in Btu per hour per square foot per unit temperature difference between the *inside surface* of the unheated parts of the enclosure and the outside air. By definition,

$$C_e=\frac{1}{\dfrac{1}{U_e}-\dfrac{1}{h_i}} \tag{5}$$

where h_i is the value of the combined (convection plus radiation) inside film coefficient as used in developing the U terms that appear on the right side of equation 2; for values of U taken from the *ASHVE Guide* or other standard references the usual value of h_i is 1.6. Equations 3, 4, and 5 are included here principally

as a matter of reference for, as will be shown, the solution of *control* problems can be attained by a graphical procedure which obviates calculating C_e or U_e. The value of the equivalent overall coefficient of heat transfer, U_e, has so little effect on the optimum minimum air temperature, t_a, that for practical purposes it may be ignored as a variable in the graphical solution of the control problem, for the normal range of structures.

The Ventilation Factor, V_c.

The U_e term is the most significant single factor influencing *design* of a panel heating system since its effect on the area of the panel is greater than that of any other variable. From the standpoint of *control,* however, the factor of greatest importance is the ventilation rate since this is the term that is most influential in determining the optimum comfort inside air temperature. The magnitude of the reduction in air temperature below 70° F for optimum comfort increases more rapidly with ventilation rate than with panel surface temperature, or U_e, or reduced outside air temperature; thus from the standpoint of control nothing is more important than an accurate evaluation of the rate at which cold outside air is introduced into the enclosure.

For convection heating systems the ventilation term of primary importance is the weight of air introduced per unit time. The older method was to estimate the volumetric air changes occurring in one hour, whereas the method recommended in recent editions of the *ASHVE Guide* is to determine the volume of infiltrating air (unless the system is one in which mechanical ventilation is used) and change this to a weight basis. Differences between the infiltration and air change methods frequently amount to from 300% to 500% for residences, but the influence of ventilation air on the total load, in convection heating, is in any event so small as not to permit of introducing a serious discrepancy in the design. In the case of panel heating, however, the situation is very different and the unduly conservative results of the air change method may be sufficient to completely alter the control characteristics of the system in question. This point cannot be overemphasized. (1) For the designer working with a convection type heating system the error due to use of the air change method is likely to be negligible and conservative. (2) For the designer working with a panel heating system the error is likely to be significant, but again conservative. But (3) for the engineer who is concerned with provision of adequate controls for a panel heating system the error is more than likely to be serious and non-conservative. Provision of satisfactory controls for any panel heating system is predicated on an accurate determination of the minimum optimum comfort air temperature that is ever likely to occur for that system; minimum air temperature is in large measure determined by ventilation rate; hence it is essential that the ventilation factor be correctly and accurately evaluated.

Of the two methods most commonly used for determining the quantity of outside air passing through an enclosure the air change method is simple but hopelessly inaccurate whereas the infiltration method (see *Guide* for details) is accurate but relatively tedious. It would seem, therefore, that some method is required which does not involve reference to a handbook (as does the infiltration method), but which is yet of greater accuracy than the rough estimate that corresponds to use of the air change method. To assist the engineer whose primary interest is in controls the following procedure is recommended and it is suggested, as a matter of convenience, that he use the formula (Equation 6) in all problems for which he must (in order to use the graphs) evaluate the ventilation factor.

The volume of air infiltrating into a room depends on the number and location of the windows as well as their size and construction, and on the outside air velocity. Since the values of the overall coefficient, U_e, that are commonly used in heat transfer calculations are based on an assumed outside wind velocity of 15 mph, it is customary to assume the same wind velocity in evaluating infiltration rates. Infiltration is usually expressed in cubic feet per hour per lineal foot of window or door crack and the rate obviously would be expected to vary inversely as the tightness of the opening. According to the 1946 edition of the *ASHVE Guide* the worst case (for average construction) occurs when windows are of the double-hung wood sash type and are unlocked; in this event the rate of infiltration is 39 cfh per lineal foot of crack. Since all other windows have less infiltration, the value for unlocked double hung windows will be amply conservative, but since it gives a ventilation factor usually from ⅓ to ¼ of that indicated by the air change method it would also appear to be a very definite improvement on the air change procedure in determining the value of the ventilation factor. Thus for purposes of rapid practical design it is recommended that the ventilation factor for all enclosures be calculated from a formula based on the infiltration through unlocked double hung wood sash windows.

The formula is derived as follows:

1. For double hung windows the crack is calculated as twice the height plus three times the width. Taking half the total lineal feet of crack around windows and outside doors, or $C/2$, as that through which

infiltration occurs, and using the value of 39 cfh per lineal foot of crack as the basic infiltration rate, the total infiltration for the room is $39C/2$ or 19.5 C.

2. For panel heating work, however, the ventilation term of greatest significance is not the total volume, but the volume of outside air that passes over each square foot of interior room surface (including the surface of the heating panel). The total inside surface area of a room of rectangular plan and uniform ceiling height is $2(LW+WH+LH)$ where L, W, H, respectively, are the length, width, and height of the room. The ventilation factor is then

$$V_c=\frac{19.5\ C}{2(LW+WH+LH)}=\frac{10\ C}{LW+WH+LH} \qquad (6)$$

Equation 6 is valid for most types of structures under most operating conditions; and although it is not as accurate as the exact infiltration method (which is not itself free of defects), use of this equation will not lead to appreciable error in design of controls for any occupied enclosure. In determining the control requirement for a given zone, this formula may be applied to one typical room, or it may be applied to two or more representative rooms and the V_c values averaged.

If it is desired to make more precise determinations with somewhat of the convenience of Equation 6, the following generalized forms of the equation may be used with basic infiltration rates obtained from the *ASHVE Guide*.

For a room with three or four exposures, a formula based on half the total crack for the room will yield a close approximation to the results obtained by following exactly the recommendation in the *ASHVE Guide* (1946, p. 267).

$$V_c=\frac{\frac{1}{2}\ C\ q_i}{2\ (LW+WH+LH)}=\frac{\frac{1}{4}\ C\ q_i}{LW+WH+LH} \qquad (6a)$$

where C is the total crack for the room, and q_i is the basic infiltration rate in cfh per lineal foot of crack, as given in the *ASHVE Guide*.

For a room with one or two exposed walls, following the recommendation in the *ASHVE Guide*, the formula becomes

$$V_c=\frac{C'\ q_i}{2\ (LW+WH+LH)}=\frac{\frac{1}{2}\ C'\ q_i}{LW+WH+LH} \qquad (6b)$$

where C' is the total crack for the one exposed wall, or the wall with the most crack.

Finally, if mechanical ventilation is used, the ventilation factor is given by the equation

$$V_c=\frac{60\ Q}{2\ (LW+WH+LH)}\times\frac{70-t_t}{70-t_o}$$

$$=\frac{30\ Q}{LW+WH+LH}\times\frac{70-t_t}{70-t_o} \qquad (6c)$$

where Q is the total volume of outside air delivered to the room, in cfm, and t_t the temperature at which it is delivered. With untempered ventilation, $t_t=t_o$ (approximately) and the temperature ratio may be ignored.

The Three Heat Balance Equations

Having defined (by equations 3 and 6) the two fundamental terms U_e and V_c, we may now write the three heat balance equations for the idealized enclosure. Following the form given in the *ASHVE Transactions** the equation for a heat balance on the unheated surface is

$$t_e=\frac{ut_p+.8vt_a+C_evt_o}{u+.8v+C_ev} \qquad (7)$$

where u and v are the heated and unheated fractions, respectively, of the total room surface, and the equation is applicable to a room heated with panels in the wall, floor, or ceiling.

The heat balance equation written on the ventilation air passing through the room (source of equation is same reference as above) is,

$$t_a=\frac{h_put_p+.8vt_e+.018V_ct_o}{h_pu+.8v+.018V_c} \qquad (8)$$

where h_p is the film coefficient of convective heat transfer from the panel and is customarily taken as .4 for a ceiling installation, .7 for a wall panel, and 1.1 for a floor panel.

The heat balance on the subject is obtained by writing the Comfort Equation (equation 1) with the weighted average inside surface temperature substituted for t_m, giving

$$t_a=140-ut_p-vt_e \qquad (9)$$

where t_e is the surface temperature of the unheated area making up the enclosure. Equation 9, like equation 7, is valid for all installations irrespective of the location of the heating surfaces.

The Single-Equation and Graphical Method

By solving equations 7, 8, and 9 simultaneously, two of the unknowns, as t_e and t_a, can be eliminated, leaving a single equation of the generalized form,

$$u=\emptyset(t_p,\ t_o,\ C_e,\ V_c) \qquad (10)$$

where the symbol $\emptyset$ indicates that the four properties on the right side of the equation are functionally related. The unheated surface area, v, does not appear in this equation because it is a dependent variable that is fixed in value as soon as the fraction of room surface to be heated is known; thus, $u+v=1$.

All properties on the right side of equation 10 are known or arbitrarily fixed (the value of t_p usually being taken as 120° F. for wall or ceiling panels and 85° F for floor panels) so the equation can be solved

*"Trend Curves for Estimating Performance of Panel Heating Systems." *ASHVE Transactions*, Vol. 48, 1942.

to determine u and thereby complete the design of the system. Numerically, however, the task of completing a design by means of the three heat balance equations (reduced to the form indicated in equation 10) is algebraically complex and arithmetically tedious. Detailed development of the explicit single design equation is available in the literature*—and a graphical solution of that equation for simplified design of copper tube type sinuous coil panels has been published†.

DERIVATION OF THE GRAPHICAL CONTROL PROCEDURE

None of these solutions, however, is of direct value to the engineer whose primary interest is controls, since the information that he is most interested in, the optimum value of the inside air temperature for design conditions, is not given by equation 10 and is not necessarily determined during the usual course of design. *From the standpoint of design* the important thing is to find out what area of panel at what temperature is required and to so space and size the tubes or ducts that the requisite panel surface temperature will be effectively and uniformly attained; *from the standpoint of control,* however, the focal point of interest is the inside air temperature.

After completing a design, so that the value of u is known, one might substitute back into an intermediate equation and determine the inside air temperature t_a. Rather than do this, however, if the control engineer finds the design completed, he may assume that the panel to be installed will be of correct size and temperature and solve directly for t_a. Thus returning to the three basic equations (equations 7, 8, 9) and eliminating t_e and u among them leads to a single equation of the generalized form,

$$t_a = \emptyset' (t_p, t_o, C_e, V_c) \qquad (11)$$

where $\emptyset'$ indicates a functional relationship (but not the same one as in equation 10) for the four properties which appear on the right hand side. But C_e is related to U_e by equation 5, so the generalized equation 11 can be more conveniently written,

$$t_a = \emptyset'' (t_p, t_o, U_e, V_c) \qquad (12)$$

The Fundamental Control Equation

Equation 12 is the fundamental equation of interest to anyone concerned with the control of a panel heating system for optimum comfort. It is applicable to any system regardless of where the panels are located (except that the form of $\emptyset''$ varies with panel location) and regardless of how they are energized. This latter point is of very great importance for it means that the analysis of control requirements for any panel heating system will lead to exactly the same results regardless of the method used to supply energy to the face of the panel, as hot water, steam, warm air, or electricity. The *method* of control used and the requirements of the control system in overcoming thermal lag, preventing hunting, etc., will differ with different heating methods (this subject will be discussed in a later section), but the statement of the control problem as evidenced by a determination of the minimum value of the optimum inside air temperature is fixed once and for all when a solution of equation 12 is obtained.

Relationship of Optimum Inside Air Temperature to Outside Temperature

As has already been indicated, the value of t_a determined by equation 12 (or by the graphical method to be described) indicates merely the maximum reduction of the inside air temperature below 70° F which the control system should provide *when the outside air temperature is at its design (average maximum load) value,* if the air-temperature, surface-temperature relationship for optimum comfort is to be maintained. But remembering that at zero load the inside air temperature should be 70° F—that is, both air and surface temperatures at 70—it follows that for any value of outdoor temperature intermediate between 70° and design temperature the inside air temperature should be held at some corresponding value between t_a and 70. The problem then is to determine the path connecting t_a and 70° F—that is, the shape of the curve which defines room air temperature as a function of the outside air temperature.

A qualitative answer can be obtained from the three basic heat balance equations by considering an installation in which u, v, C_e, V_c have fixed values and it is desired to investigate the way in which t_a varies with t_o. Letting the capital letters of the alphabet represent numerically known terms or coefficients, equations 7, 8, and 9 can now be written,

$$t_e = At_p + Bt_a + Ct_o \qquad (7a)$$

$$t_a = Dt_p + Et_e + Ft_o \qquad (8a)$$

$$t_a = G + Ht_p + It_e \qquad (9a)$$

Solving equation 9a for t_e and eliminating t_e between equations 7a and 9a,

$$t_p = J + Kt_a + Lt_o \qquad (13)$$

Solving equation 8a for t_e and eliminating t_e between equations 8a and 9a,

$$t_p = M + Nt_a + Ot_o \qquad (14)$$

Eliminating t_p between equations 13 and 14,

$$t_a = P + t_o \qquad (15)$$

*"A Single-Equation Design Procedure for Radiant Panel Heating." *ASHVE Transactions*, Vol. 52, 1946.

†*A Graphical Design Procedure for Radiant Panel Heating*. Revere Copper and Brass, Incorporated, 1946.

which is the equation of a straight line and therefore proves that, for the assumptions underlying the heat balance equations, the relationship between inside air temperature and outside air temperature, for optimum comfort, is a very simple one. The room air temperature must be 70° when t_o is 70°; it must be t_a (as read from the graphs) when t_o is at its design value; it must increase by 10% of the scale distance from t_a to 70° for each increment of t_o amounting to 10% of the scale distance from design outdoor air temperature to 70°.

Graphical Solution of the Control Equation

The practical application of equation 12 involves a number of difficulties not least of which is the tediousness of carrying through the necessary numerical computations. When the problem is one of design—that is, determination of the required area of heating panel—no short cuts can safely be used, but when the problem is one of determining control requirements a number of simplifying conditions exist. In design the most important single factor in fixing the panel area is the equivalent overall coefficient, U_e, and variation in U_e over the range found in practical installations (from 0.05 to 0.20) can result in a 100% variation in required panel area. From the standpoint of control, however, the variation in optimum design inside air temperature as a function of U_e is fortunately very small. All other factors being equal, the same maximum variation in U_e will change t_a by less than ½° F for wall and floor panels and rarely more than 1° F for ceiling panels. Thus if a mean constant value of U_e equal to 0.12 is used in equation 12 the resulting design value of t_a will be valid for any average structure, irrespective of the type of construction or amount of insulation, with an accuracy of plus or minus ½°; this is more than adequate for practical design purposes.

A second simplification arises from the fact that a variation in panel design temperature over the entire range corresponding to current practice (from 80° F to 105° F for floor panels and from 95° F to 125° F for wall or ceiling panels) will influence the optimum inside air temperature—all other factors being the same—by less than 0.1° for wall or floor panels and less than 0.3° for ceiling panels.

Applying these simplifications to the generalized equation 12, two of the variables become constants and the equation reduces to the much simpler form,

$$t_a = \emptyset''(t_o, V_c) \qquad (12a)$$

Thus for any normal type of panel designed for use in any average structure the control engineer can accurately determine the optimum value of the inside air temperature, for design outside conditions, as a function of the outside air temperature, t_o, the ventilation factor, V_c, and the panel location (as wall, floor, or ceiling).

Fig. 3 (see p. 24) is a graphical solution of equation 12a for ceiling panels. To use the graph the control engineer must know the design outside air temperature and the ventilation factor, V_c (calculated by equation 6 or one of its variants). A solution for t_a is now obtained by entering the base of the graph at the known value of V_c, rising vertically to intersection with the radial line for the known design outside air temperature, and moving horizontally to the left to read the value of t_a from the scale at the edge of the graph; an example line (broken line) on Figure 3 demonstrates the method of use.

In using the graphical solutions the engineer must remember that the indicated values of t_a are valid only for the design value of the outside air temperature and are minimum values (maximum resetting of control point); they are of value to him in that they permit immediate determination of the need, or lack of need, for a control system that will vary the setting of the inside air temperature as a function of load. They also permit determination of the ratio of air temperature reset range to outdoor air temperature range to be provided by the control system.

From the above discussion it follows that the control engineer can always use the graphs with assurance that they will give him an answer that will permit immediate determination of the need, or lack of it, for a control system which will vary the inside air temperature as a function of load. Work with the graphs will soon show that there is very little need for such variable control in well-insulated residences with moderate window area, but that the use of large window areas in a house or the use of mechanical ventilation (as in office buildings, shops, etc.) will necessitate provision of a control system which will permit re-setting of the inside air temperature as a function of load.

THERMAL INERTIA IN RELATION TO CONTROL

When it has been determined that the reduction of room air temperature below 70° at maximum load, for optimum comfort, is (or is not) large enough to require control in terms of a reset control point, it is still necessary to estimate how the thermal inertia of the panel and of the structure will influence the control response of the installation. The apparent time lags involved depend upon the thermal capacity and resistance of panel and structure, and influence both the effectiveness of control during periods of changing load and the stability of control under a given load.

The thermal capacity of a body is defined as the product of the volume, the density, and the specific heat. It expresses the total quantity of heat that must enter or leave storage for a unit change in temperature, in Btu per degree F. The product of thermal capacity and degrees temperature change (i.e., change in mean temperature of the whole mass) is the quantity of heat that the panel, for example, must absorb or dissipate before reaching a new equilibrium.

Resistance as a thermal property is variously defined in the literature, according to its use, as the reciprocal of conductivity, of conductance, of surface conductance, etc. For the present purpose it may be taken as the reciprocal of overall transmission coefficient, for the structure, and as the reciprocal of conductance, for the panel. Exact definition, however, is hardly necessary where strictly quantitative expression is not aimed at, and in estimating the types and combinations of instruments necessary for effective and stable control of a given installation, a qualitative comparison of the time lag characteristics of panel and structure will generally be sufficient. What is important is the recognition of resistance to absorption or dissipation of heat as one of the two factors determining the delay in internal reflection of an external load change, or the delay in establishing equilibrium conditions at the panel on a change in energy supply. The greater the heat storage capacity of the panel or of the structure and the longer the time required for a given quantity of heat to enter or leave the reservoir represented by that thermal capacity, the longer will be the time lag of the panel or the structure in reaching equilibrium.

Although the individual capacities and resistances of the panel and the structure are inseparably related as parts of the overall transient characteristic of the whole *system* (*i.e.*, panel and structure taken together), their significance can perhaps the more readily be grasped if the lag characteristics of the panel and of the structure are first considered separately, in terms of the problems in which one or the other is the more conspicuous factor.

INERTIA OF THE PANEL

Ignoring for the moment the characteristics of the structure, consider by way of illustration the behavior of a typical hot-water concrete floor panel in response to a marked change in heating load.

Assuming that the panel measures 20 by 20 feet, consists of 4 inches of concrete effectively insulated on the under side, and is rated at 35 Btu per square foot per hour maximum energy output, under design conditions the calculated rate of heat loss from this room must be $20 \times 20 \times 35 = 14{,}000$ Btu per hour. Assume further that a fixed air-temperature control point of 67° F is used. In order to maintain an 85° F panel surface temperature the mean water temperature (assuming the use of embedded coils on spacings corresponding to standard practice) will be of the order of 110° F. Suppose now that the outside air temperature increases sufficiently to reduce the heating load from 100% to 90% of maximum. Such a change could readily and rapidly occur as a result of solar gain due to dispersion of clouds. How much time would be required to establish the new equilibrium temperature in the slab?

With normal construction procedures the embedded coils would be close to the surface of the slab, and since the slab is well insulated it can be assumed that the mean slab temperature will be approximately 5° lower than the mean temperature of the water in the coils. The initial temperature difference between slab surface and room air is $85\text{-}67 = 18°$, and since the load is to be reduced by 10%, a reasonable first approximation to the new equilibrium value of slab surface temperature would be 85-0.1(18) = 83.2° F; and a reasonable approximation to the new equilibrium value of the mean water temperature is ~~110-0.1(110-67)~~ 83.2 + 0.9 (110-85) $= 105.7°$ F. Thus the change in mean temperature of the slab corresponding to a 10% reduction in load is of the order of 4.3° F (110-105.7°).

The thermal capacity of the slab is its volume times its density times its specific heat. The density and specific heat of the slab will be taken as 130 lb. per cubic foot and 0.5 Btu per lb per deg F respectively. Then

Thermal capacity of slab $= 20 \times 20 \times \frac{1}{3} \times 130 \times 0.5 = 8700$ Btu per degree.

Hence the quantity of heat that must leave storage in the slab before the new equilibrium conditions will be established (assuming no heat input to the slab during the transient period) would be

$$8700 \times 4.3 = 37{,}400 \text{ Btu.}$$

Note, therefore, that even if circulation of water through the slab were stopped it would still require approximately

$$\frac{37{,}400}{14{,}000} = 2.7 \text{ hours}$$

for the room to reach equilibrium. This result is decidedly conservative in view of the fact that most panel heating systems operate with continuous flow of the heating medium so that in this example, even if the boiler were shut down, the circulating water would continue to lose heat to the slab until all the water in the system had been reduced to the slab temperature. Moreover, in these calculations the drop

in output of the panel with the decrease in surface temperature has been neglected. Thus this simple example, corresponding to conditions not different from those found in many actual installations, shows that a thermally "heavy" panel (regardless of the heating medium used) cannot rapidly respond to load changes, and responsibility for failure of such a panel to remain under control must be attributed to the construction rather than to the control system.

But lag in response varies also with thermal resistance. In order for a panel to deliver heat to a room, the energy source within the panel must necessarily be maintained at a higher temperature than the panel surface. A ceiling panel operating at 100° F, for example, may require water or air at a mean temperature of the order of 140° F. Suppose, now, that a very sudden decrease in load is imposed within the house, so that the air temperature starts to rise. Such a situation will frequently occur in a house with large window areas. In this event the rate of heat loss from the panel surface will be significantly reduced and even though the control system instantly and completely shuts off the supply of energy to the panel its surface temperature may none-the-less actually *rise*. At first thought such a phenomenon is baffling, and in many cases the "flash" of panel surface temperature has been incorrectly attributed to faulty control. That controls are not responsible should be self-evident. In the extreme case, if the load were suddenly decreased to zero the panel surface temperature might rise to a value not much different from the temperature of the element within the panel. In practice, however, the sudden load change is rarely more than 20% of maximum and the flash of surface temperature is only a few degrees and lasts only a short time.

Avoidance of "flash" requires that two conditions be met in the design. First, the resistance between the heating element and the panel surface should be kept as small as possible in order to reduce to a minimum the temperature difference needed to maintain the panel at rated capacity. If the resistance is low the fluid temperature (or resistance wire temperature of an electric panel) will be only a few degrees above the surface temperature and "flash" will not take place. Secondly, by reducing the thermal capacity, the amount of energy which must enter or leave the panel to re-establish equilibrium will be reduced and the duration of "flash"—if it does occur—will be shortened.

Note that, even if "flash" does not occur, high thermal resistance and capacity in the panel make for "hunting" by a room thermostat used as the controller. The thermostat cannot change the rate of supply to the panel until the panel *surface* temperature has changed enough to bring about a measurable change in the air temperature at the thermostat. But before this happens, the heat stored in the panel will be increased—or decreased—beyond the level required to satisfy the heat demand of the thermostat, and thus the "flywheel" effect of the panel's inertia will cause the air and surface temperatures in the room to rise or fall very noticeably after the thermostat has indicated that the correction originally called for has been achieved.

INERTIA OF THE STRUCTURE

The sample calculations in the preceding section showed that an ordinary type of floor panel, on a sudden change in weather conditions amounting to a 10% decrease from maximum load, would require under the most favorable conditions approximately 3 hours to reach equilibrium. From the standpoint of means of adequate control just what does this imply? The answer would naturally be sought in terms of load change anticipation by means of a control element sensitive to outside temperature (or environment, to include solar and wind effects). But assume, for example, that the thermal inertia of the house is negligible (as well it might be for some of the proposed metal wall structures with low density insulation). In this case the imposition of a 10% change in load would be reflected in a matter of minutes in the inside air temperature, and the outside control element would not serve any effective purpose: a room thermostat would sense the load almost as soon. If, on the other hand, the structure has considerable thermal inertia, there will be a delay between imposition of external load and appearance of that load within the structure. As the thermal inertia of the house increases it follows that the effectiveness of anticipatory control likewise increases. Carried to the extreme, however, a structure of practically infinite thermal capacity would (like a house of zero capacity) be unable to use anticipation, but for a different reason—the fact that the time between imposition of load and its internal appearance would be too great to permit allocating to the anticipatory control element any degree of control large enough to be of practical significance. Thus it is evident that the anticipatory feature introduced by use of an outside temperature sensitive element can be of advantage only when the lag of the structure (as determined by thermal capacity and resistance) is neither too small nor too great.

But the capacity and resistance of the structure influence also the response of the whole system to changes in heat input. Consider the case, described early in this chapter, in which the air temperature was maintained approximately constant but room

conditions rarely satisfied the definition of comfort (even under constant load) because the surface temperature tended to oscillate or "hunt". This can now be explained by visualizing a room in which the thermal capacity of the structure is relatively great and the resistance relatively low. In such a room the influence on air temperature of a change in panel surface temperature will be slight because a considerable part of any additional energy delivered to the room goes immediately into storage in the structure. Hence a relatively long time interval (and a relatively large change in panel surface temperature) will occur before the effect of altered panel surface temperature is detected by the room thermostat. Under these conditions, even though the external load remains practically constant, a cycling of panel surface temperature may occur which will be serious both in amplitude and in the time interval. For the residence discussed it was found that—at constant load—the air temperature varied approximately 1° over a three hour period while the panel surface temperature was completing one cycle whose amplitude was close to 20° F. From the comfort standpoint assume that the air temperature should be—and is—constant at 65° F. The required average surface temperature of the room would then be 75° F and this might typically necessitate a panel surface temperature of 110° F. If, under these conditions, the panel surface temperature cycles between 100° and 120° it is evident that the overall average surface temperature of the room will likewise cycle, through a smaller amplitude, yet widely enough so that comfort conditions will not exist. Thus in the room described the variation was so great that the occupants were too cold for approximately one hour, reasonably comfortable for one hour, and too warm for the third hour.

Solution of a problem of this type obviously requires provision of a control system which will permit modulation of the energy supply to the panel and will restrict the energy increment which is immediately available to a small range on either side of the normal quantity corresponding to the given load. For this purpose, an outside control is used effectively as a means of selecting the basic energy rate whereas the variation in energy that can be brought about by the room thermostat is limited. Note particularly that the function of an outside controller when used for this purpose is in no sense the same as its function when used to provide anticipation of load. Even in a system where anticipation is not feasible (because of very small or very large structural thermal capacity) an outside control element may still be required to prevent cycling—at constant load—of the panel surface temperature.

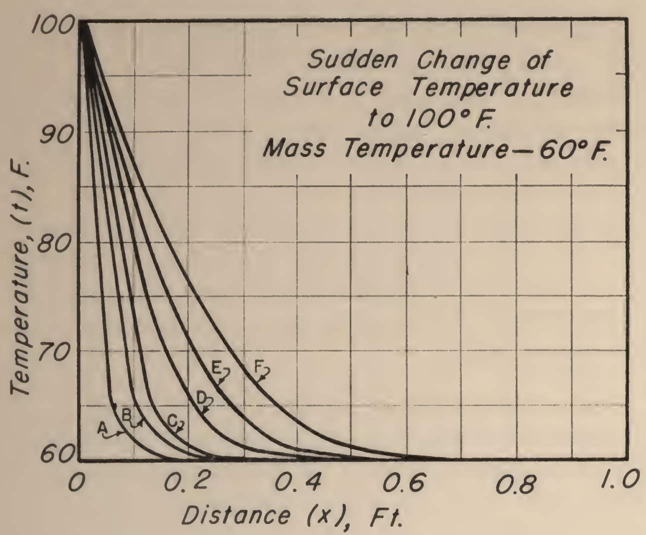
RELATIONSHIP OF INERTIAS OF PANEL AND STRUCTURE

From the preceding discussion it follows that the factor of greatest importance from the standpoint of control is the relationship between the inertia factors of the panel and of the structure rather than the absolute value of either. Practically, the criterion of control effectiveness is whether the lag in reproduction of an externally imposed load within the house is equal to or greater than the lag in establishing equilibrium at the panel.

If this *is* the case, anticipatory control can be used to great advantage, but if this is not true the value of the anticipatory feature is less pronounced. Note that when both lags are equal, anticipation alone will suffice; when the lag of the structure exceeds the lag of the panel, the outside control will be effective if its function is to select the basic energy supply rate and if it is qualified by an inside air temperature sensitive element; but if the lag of the panel exceeds the lag of the structure, the anticipatory control will be incapable of providing sufficient "warning" to permit the inside thermostat to retain control.

A concrete floor slab, when used as a panel, always poses a control problem, but the seriousness of the problem varies inversely as the thermal lag of the structure. Thus a 4″ concrete floor used as a heating panel in a house with large glass area (such as would be found in a locality where solar heating is extensively used) would present control problems that might be insurmountable, whereas a similar slab in a concrete house of orthodox construction would be a simple matter to control.

From the foregoing discussion the generalization that seems most evident is that, for the sake of adequate control, every effort should be made by the designer to see that the heated panel used in any structure does not differ materially in thermal capacity and resistance from the exterior wall of the house; if they differ materially, it is vitally important that the thermal capacity and resistance of the panel be small as compared with those of the structure. Thus control of a heated concrete floor slab in a concrete house is a simple matter, and control of a frame type (metal lath and plaster) panel in a frame house is also relatively simple; control of a frame type panel in a concrete house would therefore not constitute a difficult problem. But satisfactory control of a concrete panel in a frame house would, in all probability, be practically impossible. This is particularly true if inadequate insulation is provided below the slab since, in this event, the moist earth is in direct contact with



$$t = t_s + (t_i - t_s)\frac{2}{\sqrt{\pi}}\int_{n=0}^{n=\frac{x}{2\sqrt{aT}}} e^{-n^2}\,dn$$

t = Temperature at x

t_s = Surface Temperature

t_i = Mass Temperature

x = Distance, Feet

T = Time, Hours

a = Thermal Diffusivity = $\frac{k}{cp}$

k = Thermal Conductivity

cp = Specific Heat of Unit Volume

c = Btu per lb. per F.

p = lb per cu. ft.

Fig. 1—Temperature Gradients in Fill Under Panel.

the slab and because moist earth has high conductivity (as well as high thermal capacity), the effective thermal capacity of the slab is enormously increased; thus the "earth" effect is to increase the apparent time lag of response of the slab. The magnitude of the effect is strikingly indicated in the accompanying table and curves*, derived by calculating, with standard mathematical methods from established physical constants, the results of an assumed sudden temperature change at the surface of the fill underlying a floor slab. In Fig. 1 are plotted the temperature gradients through the upper layers of the fill after various time intervals (curves A to F), and Table 1 lists the time intervals required for each of these temperature-distance curves to be attained in moist clay, in soil taken as having the minimum conductivity listed in the *ASHVE Guide*, and in air-cooled blast furnace slag laid on a moisture barrier. Observe, in Table 1, that the clay fill requires scarcely more than half an hour to attain the temperature gradient (curve F) for which 7 hours are required in the slag fill. Similarly, in Fig. 2 (total heat transmission through the surface of the fill as a function of time) it may be observed that in 1 hour the clay will absorb approximately five times as much heat as the slag. Granted that the assumed initial conditions are more extreme than would occur in practice, these results nevertheless underscore the likelihood of an increased lag in response to a demand for more heat. And the reminder that the maximum thermal conductivity for soil is still only from 1/12 to ⅛ that of ordinary concrete† may reinforce the suggestion advanced in the article cited, that under certain conditions, even though the control system has shut off the energy supply of the panel, reverse flow of heat from soil to panel may occur, to prolong the period of overheating.

Table 1—Time Values in Hours for Fills

Curve (See Fig. 1)	Clay	Soil (Minimum)	Slag
A	3/160	⅛	¼
B	3/80	¼	½
C	3/40	½	1
D	27/160	1⅛	2¼
E	3/10	2	4
F	21/40	3½	7

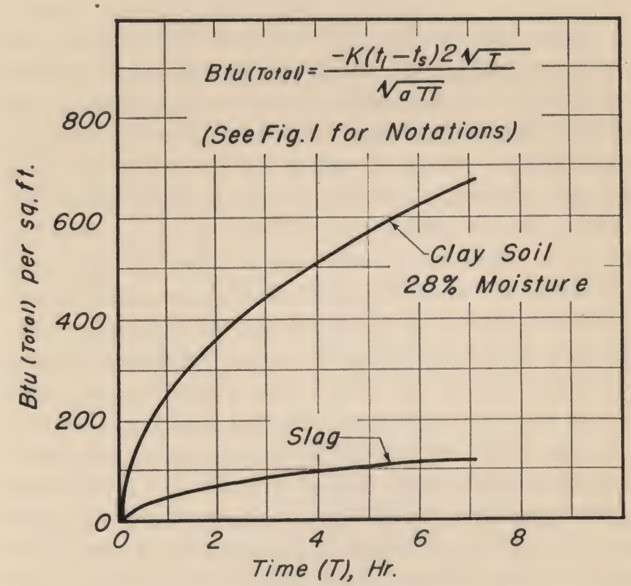


Fig. 2—Total Heat Transmission Through Surface of Fill as a Function of Elapsed Time.

*From J. E. Peterson, "Improved Floor Construction for Radiant Heating System." *Heating, Piping, and Air Conditioning*, July 1946. Reproduced by permission.

†For moist clay, Peterson, Table 1. For concrete, *ASHVE Guide*, 1946, p. 117.

SUMMARY OF THEORY

For convenient review, the main points that have been developed may be summarized in the following numbered statements, with brief additional comment where necessary.

1. *The purpose of any automatic space-heating system, regardless of type, is to maintain comfortable temperatures indoors during cold weather.*

2. *The average lightly-clothed adult at rest is comfortable when the temperature of the surrounding air and the average overall temperature of the surrounding surfaces are both at 70° F., or so related that*

$$t_a = 140 - t_m \quad (1)$$

3. *Whereas convection heating tends to permit the overall surface temperature to fall farther below room air temperature as the heating load increases, panel heating tends to raise the overall average surface temperature farther above air temperature.*

4. *For ideal comfort, air temperature should be increased with load in convection heating, but decreased with load in panel heating.*

5. *An apparent exception to the foregoing rule is found in "spot" panel systems for sparsely occupied factory buildings, where air temperature and panel surface temperature may be separately controlled at fixed values through all of the range of outdoor temperature below a critical value for which the waste heat from the panels is sufficient to carry the convective heating load.*

6. *In general space heating with a panel system, however, satisfactory performance depends upon a control system designed to maintain the proper relationship between air and surface temperatures in the enclosure.*

Conditions satisfactory for comfort cannot generally be expected, if it is assumed that the purpose of the control system is to maintain a given, constant room air temperature. Remembering that the departure of air temperature and average surface temperature from 70° F under any given load conditions should be equal and opposite, it appears that a control system designed to maintain the air temperature at the value required by the characteristics of the installation and the comfort equation, at a given load, will at the same time maintain the proper overall surface temperature. Thus the air temperature serves as an index of the temperature relationship.

7. *How necessary variable air temperature control is for practical comfort is determined by the magnitude of the calculated reduction of air temperature at design load.*

If the reduction is small—say 1 or 2 degrees—constant air temperature control will provide an acceptable approximation to ideal comfort standards. If it is larger, provision should be made for reducing the room air temperature as a linear function of outside air temperature.

8. *The range of room air temperature for optimum comfort ($70-t_a$) may readily be determined from design data, one equation, and the proper graph.*

With the design completed, the control engineer need only calculate the ventilation factor, V_c, for a typical room, or the average for two or more representative rooms, in each zone, select the graph for ceiling, wall, or floor panel, and read the design optimum air temperature directly from the graph. The graphs may be used with assurance, since over-sizing of the panel, the most likely design error, increases the need for reduction of the air temperature at design load.

Experience will soon make it possible to estimate the need for variable air temperature control, but for most installations the added certainty gained by determining the value of t_a from the graphs will repay the slight investment of time and effort.

9. *The problem of effective and stable control, as distinguished from the required performance of the system under control, resolves itself into a question of thermal inertia or time lag.*

10. *The time lag problem is accentuated in panel heating by the comparatively large mass of the panel and by its location in the heated space.*

11. *In general, thermal inertia or time lag varies with thermal capacity and thermal resistance.*

For the common run of installations, thermal resistance corresponds to thermal capacity sufficiently to permit a qualitative estimate of thermal inertia in terms of apparent relative mass—*i.e.*, a "light" or "heavy" panel, in a "light" or "heavy" structure. Occasionally, a discrepancy between capacity and resistance, particularly in the structure, may introduce a peculiar problem of stability.

12. *Large capacity and resistance in the panel mean slow response to controls.*

13. *Large capacity and resistance in the structure mean delay in reflection of external load changes within the structure.*

Within the range commonly encountered in practice, such structural lag is advantageous to control, since it permits use of an outside control for anticipation of load changes.

14. *Large capacity and relatively low resistance in the structure may cause conspicuous instability of control from a room thermostat.*

The indicated solution is to use an outside control element to modulate the basic energy supply rate to the panel in accordance with load, and to permit the room air temperature control to make relatively minor adjustments in the basic rate of energy supply.

15. *An outside control device is thus often desirable, first as a means of anticipating load changes, and second as a means of reducing the control of the room air thermostat and thereby increasing the stability of the system.*

The effectiveness of outside control for purposes of anticipation (with a heavy panel) is dependent on the relative thermal inertia of the structure and is small if the structure has either very low or very high thermal inertia. Fortunately, however, most standard buildings have thermal capacities falling well within the range of effective anticipatory control (provided the mass of the panel is reasonably well matched to that of the structure). Exceptions are heavy masonry windowless factory buildings and light residential construction with large window areas. As a means of preventing over-control by a room thermostat, on the other hand, outside control will be effective regardless of the thermal capacity of the structure, but will increase in effectiveness as the resistance of the structure decreases.

16. *In general, effective and stable control may be attained by some means, only if the lag of the panel is equal to or less than the lag of the structure.*

From the standpoint of design as the limiting factor for effective and stable control, the size and weight of the heating panel should always be kept as small as possible and—in particular—heavy panel construction should be avoided in a light structure; for heavy masonry structures the weight of panel is not of great control importance since, in such cases, the ratio of the thermal capacity of the panel to that of the structure is, in any event, favorably small.

17. From the practical standpoint, in terms of the commonest types of installations, the selection of controls may be summarized thus: *usually a light panel may be controlled by a room thermostat (with control point reset when necessary); a heavy panel, by an outside controller (qualified when necessary by an inside controller).*

In all but the exceptional cases, the panel will either be light enough to be amenable to room thermostat control or heavy enough to require control by a weather-sensitive element, regardless of the thermal inertia of the structure. In the case of a heavy panel in a light structure, difficult as it may be to achieve fully satisfactory control, an outside controller will at least take advantage of what structural lag is available to compensate for the lag of the panel.

As has been previously indicated, the lowering of room air temperature with outside temperature will be unnecessary, by any but the strictest standard of comfort, except where the number or construction of windows or the use of untempered mechanical ventilation introduces a comparatively large ventilation load. When necessary, this variable air temperature may be provided, with a light panel, by resetting the control point of the room thermostat. With a heavy panel, accurately designed and operating under command of an outside controller properly adjusted, the variation of air temperature may be provided automatically.

Applications of Controls to Panel Heating

OUTLINE OF CONTROL PROCEDURE

The procedure for determining the control point requirement of a panel installation may for convenience be summarized in outline as follows:

1. *Known Factors*

Data presumably known from the panel design, or obtainable from the architect's prints or by observation at the site include: (a) panel location; (b) design panel surface temperature; (c) design outside air temperature, t_o; (d) room dimensions—L, W, H; (e) number, size and type of windows and doors; (f) mechanical ventilation, if used.

2. *Ventilation Factor, V_c*

Calculate the ventilation factor, V_c, in cfh per square foot of room surface, by equation 6,

$$V_c = \frac{10\,C}{LW+WH+LH}$$

where C is the total crack around all windows and exposed doors in the room.

For a room with mechanical ventilation, the ventilation factor is given by equation 6c,

$$V_c = \frac{30\,Q}{LW+WH+LH} \times \frac{70-t_t}{70-t_o}$$

where Q is the volume of outside air (in cfm) delivered to the room, at temperature t_t (for tempered ventilation).

The room selected should be typical of the zone; or two or more representative rooms in the zone may be used, and the V_c values averaged.

Where a closer approximation is desired to the results that would be obtained by following exactly the practice recommended in the *ASHVE Guide*, equations 6a and 6b may be used, for rooms with three or more exposures, and rooms with one or two exposures, respectively.

$$V_c = \frac{\tfrac{1}{4}\,C\,q_i}{LW+WH+LH} \qquad (6a)$$

where C is the total crack for the room, and q_i is the basic infiltration rate in cfh per lineal foot of crack, for the given type of window (and wind velocity).

$$V_c = \frac{\tfrac{1}{2}\,C'\,q_i}{LW+WH+LH} \qquad (6b)$$

where C' is the total crack for the one exposed wall, or for the wall with most crack (two exposures).

3. *Design Inside Air Temperature, t_a*

Select the proper graph according to the panel location (ceiling, wall or floor) and proceed as follows (see broken example line in Figs. 3, 4, and 5):

a. Enter the graph at the calculated value of V_c on the base scale.

b. Rise vertically to intersect the radial line for the known design value of t_o.

c. Project this point horizontally to the left, to intersect the t_a scale, and read the value of t_a.

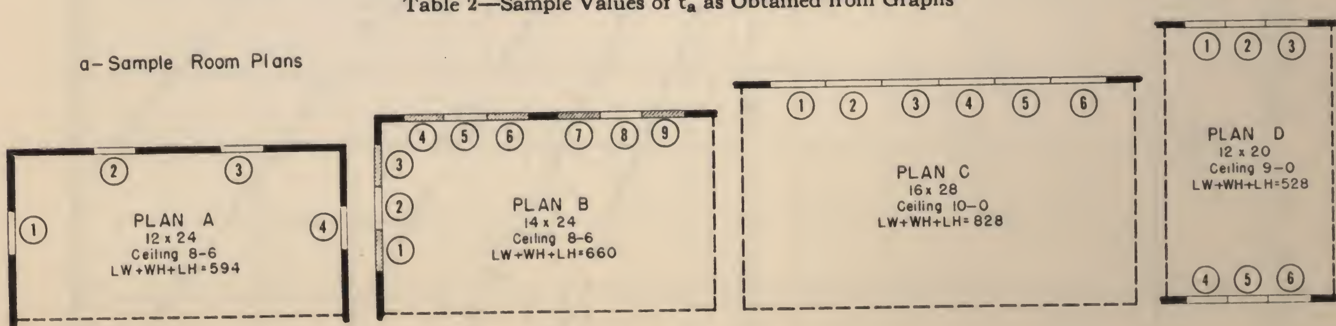
d. If this value is within 1 or 2 degrees of 70° F it may be assumed that no special provision need be made for automatic reduction of the inside air temperature with increasing load. If larger, the reduction should be allowed for in the specification of the control system.

Table 2 lists values of t_a for various representative combinations of room dimensions, window types and sizes, and design outside temperature. The examples in the main table (2c) are identified by reference to the floor plan sketches (2a) and the table of window types (2b), and the right-hand column indicates the formula used for calculating V_c for each.

These examples are presented as a matter of interest. Examination of them will give a concrete indication of what to expect on typical installations.

Table 2—Sample Values of t_a as Obtained from Graphs

a—Sample Room Plans



b—Window Types and Infiltration Rates†

Code	Description	Basic Inf'n Rate*
a	Double-hung wood sash unlocked, not weatherstripped, average construction	39
b	Ditto, weatherstripped	24
c	D-hung metal, weatherstripped, unlocked	32
d	Rolled section heavy casement projected 1/64 in. crack	18
e	Industrial pivoted, 1/16 in. crack	176

*Wind 15 mph.

c—Values of Vc and ta

Plan	Windows No.	Windows Type (table b)	Windows Size	C (C')	Vc	ta, to = 20° F. Floor	ta, to = 20° F. Wall	ta, to = 20° F. Clng	ta, 0° F. Wall	ta, 0° F. Clng	ta, −20° F. Wall	ta, −20° F. Clng	Vc by Eq'n No.
A	4	a	3 x 5	76	1.3	69.2	69.3	69.1	69	68.6	68.7	68.1	6
A	4	b	3 x 5	76	0.75	69.5	69.6	69.4	69.4	69.3	69.2	69.1	6a
B	9	a	3 x 5	(114)	3.5	67.8	68.1	67.6	67.4	66.4	66.7	65.2	6b
B	3	b	3 x 5	(38)	0.7	69.6	69.7	69.5	69.5	69.4	69.3	69.2	6b
C	6	d	4 x 6	(156)	1.7	69	69.1	68.9	68.8	68.3	68.5	67.7	6b
C	6	c	4 x 5	(132)	2.55	68.5	68.7	68.3	68.2	67.5	67.7	66.5	6b
C	6	e	3 x 3	(72)	7.6	65.3	65.9	64.8	64.4	62.3	62.8	—	6b
D	6	d	3 x 5	(63)	1.0	69.4	69.5	69.3	69.3	69	69.1	68.6	6b

†Abstracted by Permission, Table 2 Chapter 8, ***Heating Ventilating Air Conditioning Guide*** 1946.

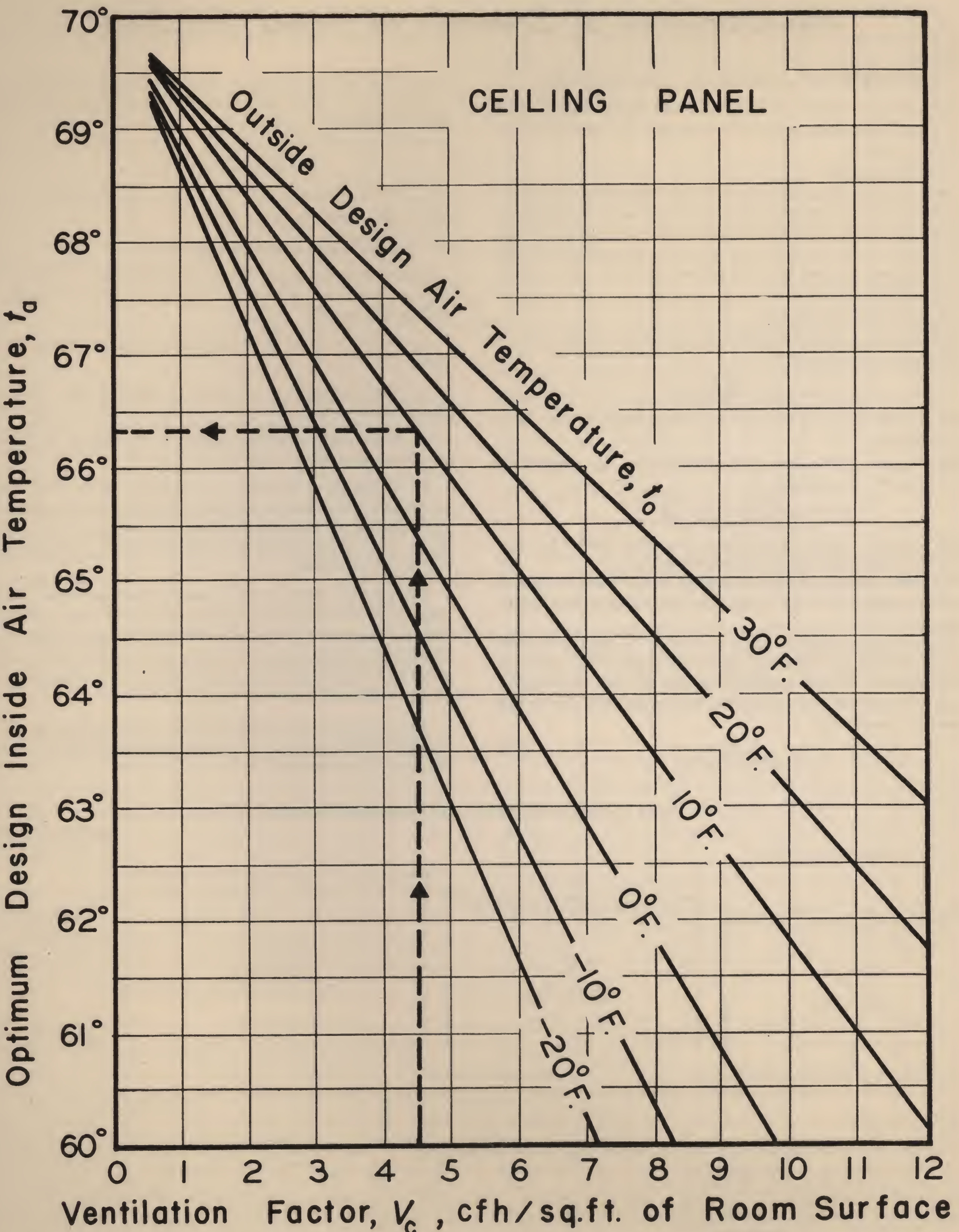


Fig. 3—Ceiling Panel Graph. Valid For Design Panel Surface Temperatures From 95 F To 125 F And For All Ordinary Structures (U_e From 0.05 To 0.20)

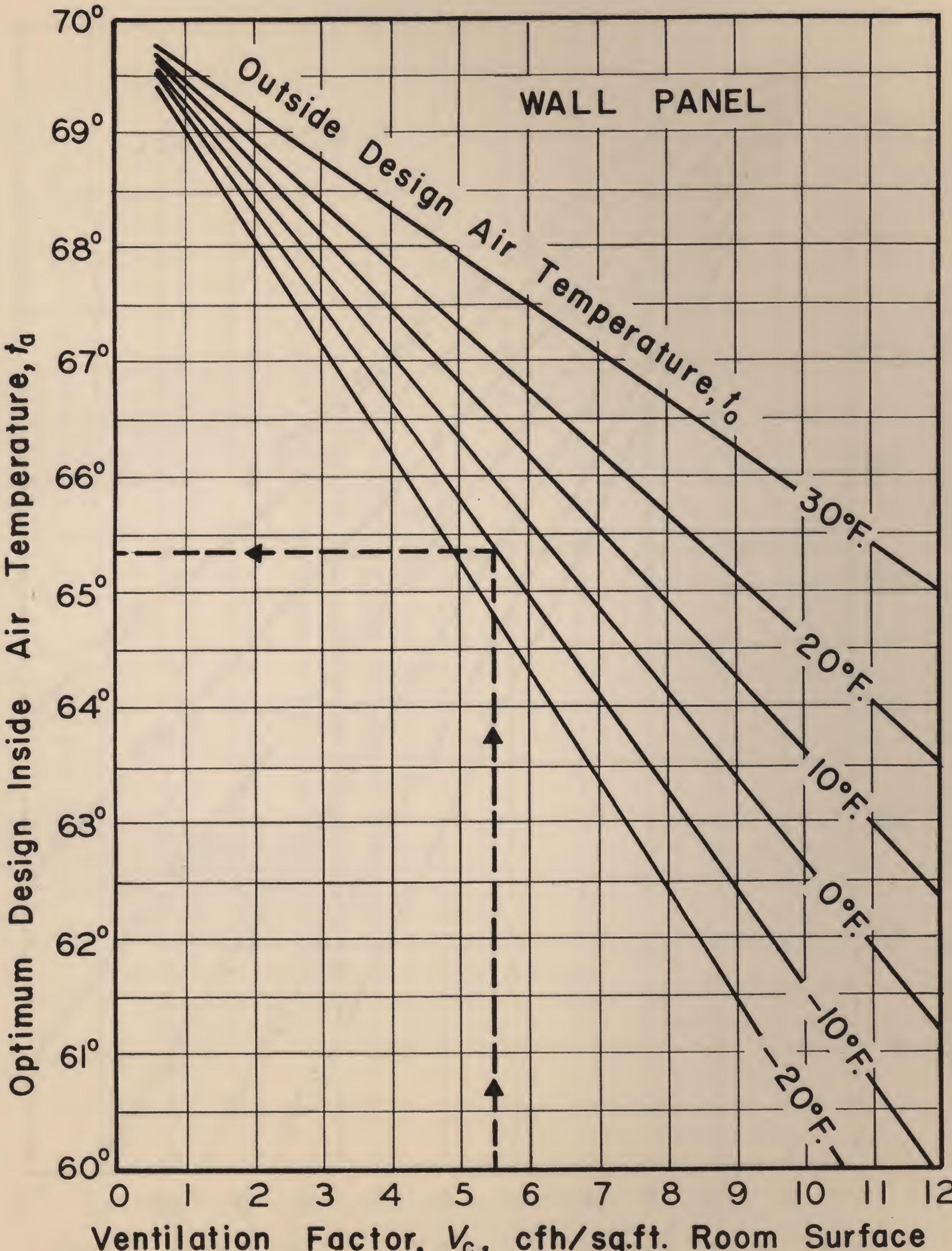


Fig. 4—Wall Panel Graph. Valid For Design Panel Surface Temperatures From 95 F To 125 F And For All Ordinary Structures (U_e From 0.05 To 0.20)

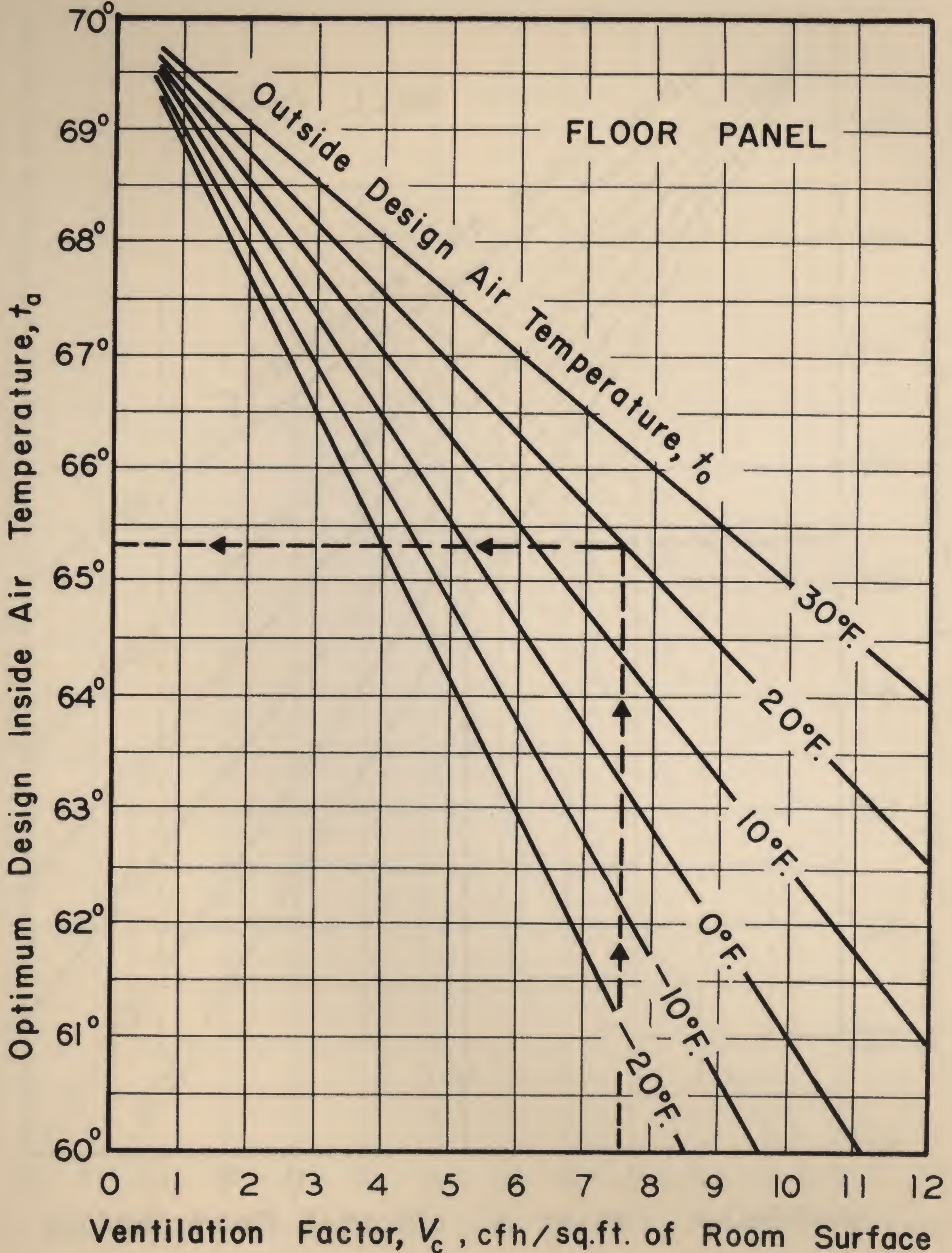


Fig. 5—Floor Panel Graph. Valid For Design Panel Surface Temperatures From 80 F to 105 F And For All Ordinary Structures (U_e From 0.05 To 0.20)

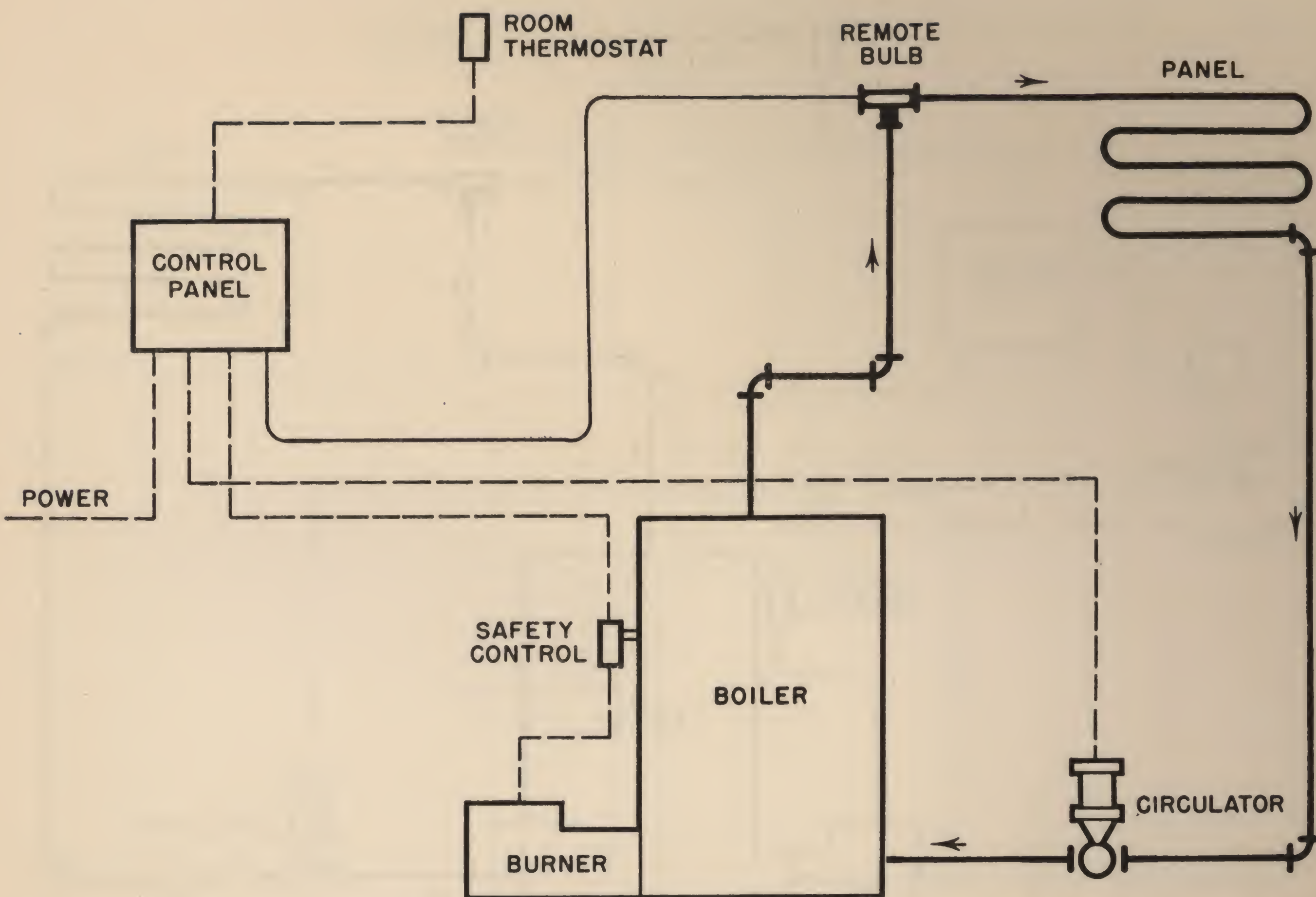


Fig. 6—Single Zone Electric—Forced Hot Water—Without Indirect Heater—Light Panel*

The room thermostat measures a change in the heating load and transmits a signal to the control panel, which accordingly raises or lowers the control point of the temperature controller whose temperature-sensitive bulb is located in the panel supply water (or in the boiler).

The temperature controller operates the burner to provide the proper water temperature as measured by the temperature-sensitive bulb.

The circulator operates continuously except when no heat is required.

The safety control will stop the burner whenever the boiler water reaches the maximum safe or desirable temperature.

*NOTE: The arrangement of all controls, panels, etc., in this and the following diagrams is purely schematic and is not intended to represent actual layouts for installation purposes.

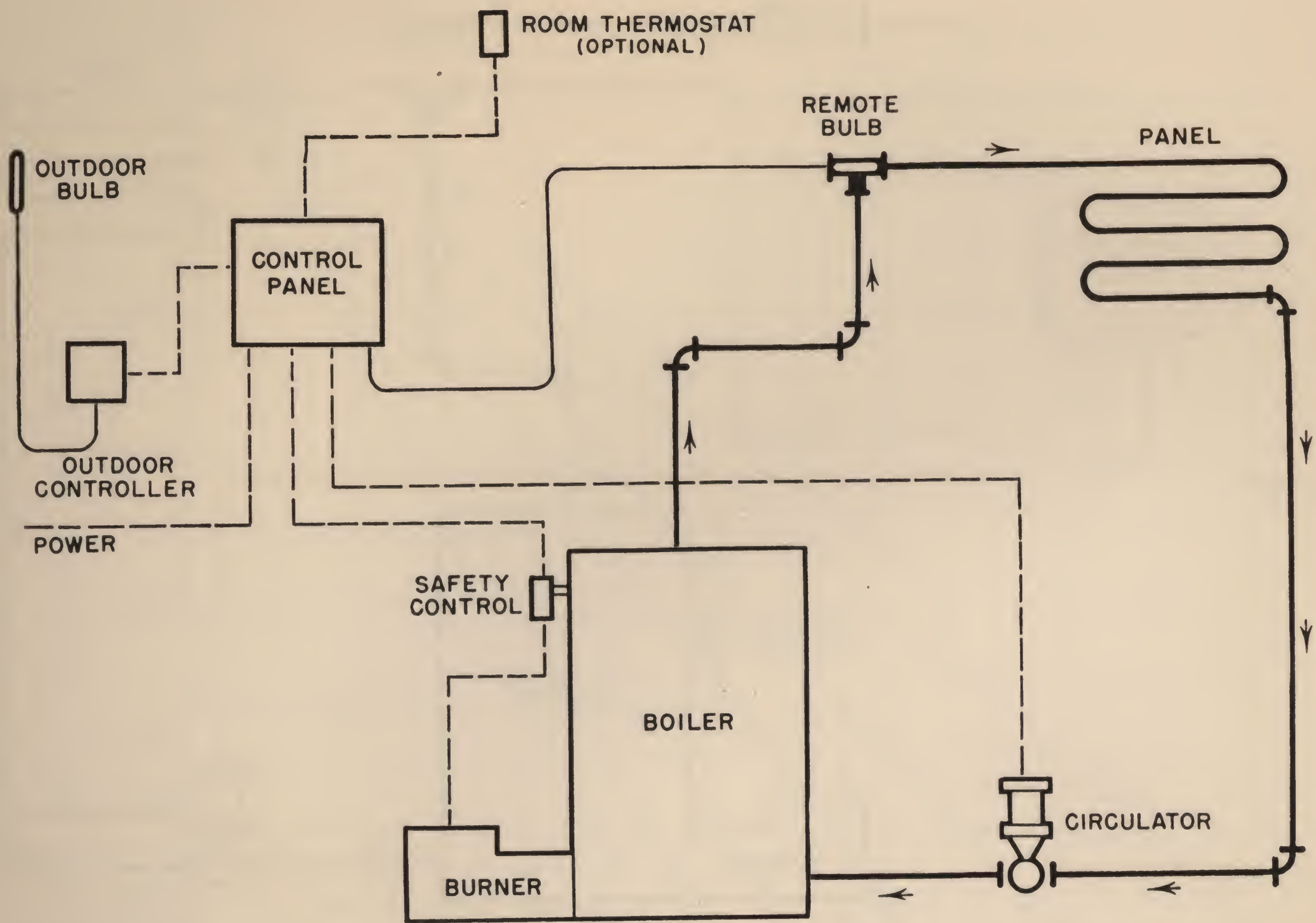


Fig. 7—Single Zone Electric—Forced Hot Water—Without Indirect Heater—Heavy Panel

The outdoor controller measures any change in the heating load as represented by the outdoor temperature. This change in load, transmitted to the control panel, causes a corresponding change in the setting of a temperature controller whose temperature-sensitive bulb is located in the panel supply water (or in the boiler).

The temperature controller operates the burner to provide the required water temperature as measured by the temperature-sensitive bulb.

The thermostat, if used, senses changes in internal heat loads caused by occupancy, solar gain, etc., and acts as a limit control.

The circulator operates continuously except when no heat is required.

The safety control prevents excessive boiler water temperature.

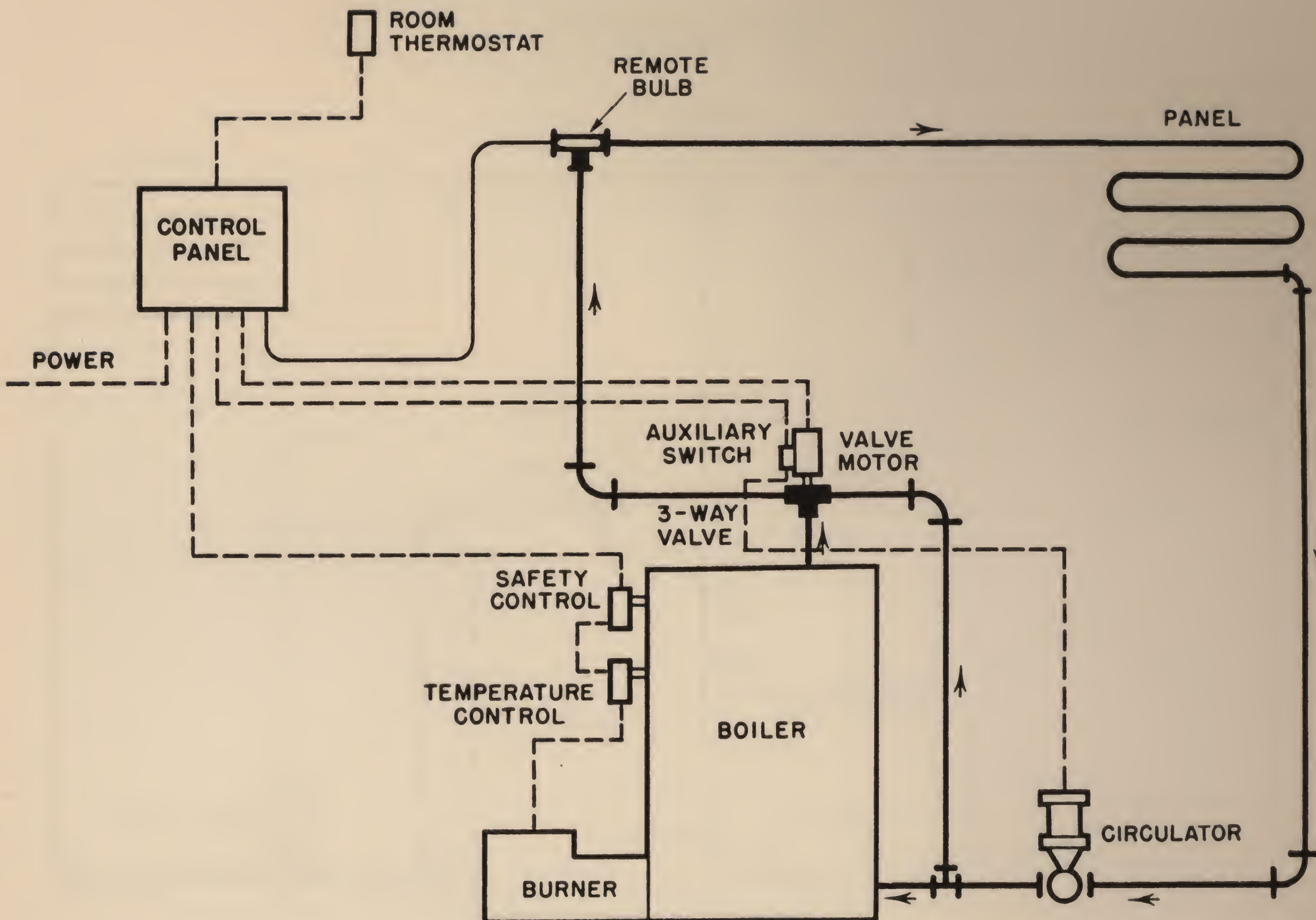


Fig. 8—Single Zone Electric—Forced Hot Water—With Indirect Heater—Light Panel

The room thermostat measures any change in the heating load and transmits the change to the control panel, which accordingly adjusts the control point of a temperature controller whose temperature-sensitive bulb is located in the supply water to the radiant panel.

The temperature controller repositions the three-way valve, which mixes hot boiler water and cooler return water from the bypass to provide a mixture of the proper temperature to satisfy the temperature-controller setting.

The burner is controlled by an Aquastat (immersion temperature controller) to maintain the necessary temperature in the boiler for the indirect water heater.

The circulator is shut down by the auxiliary switch whenever the three-way valve is positioned for zero heating (fully open bypass).

The safety control will shut off the burner if for any reason the boiler water temperature reaches the maximum safe level.

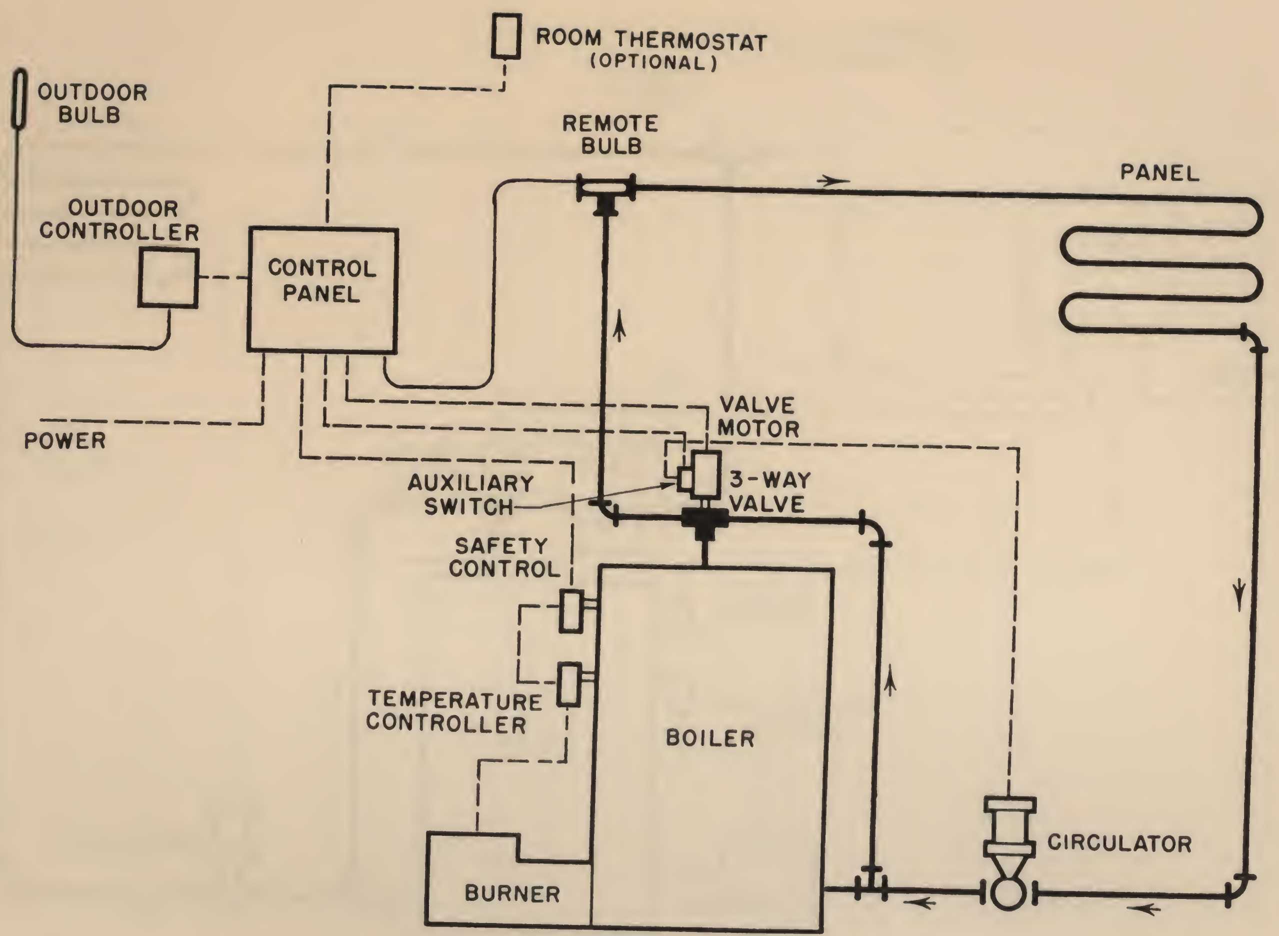


Fig. 9—Single Zone Electric—Forced Hot Water—With Indirect Heater—Heavy Panel

The outdoor controller measures changes in the heating load as represented by outdoor temperatures and transmits them to the control panel, which makes corresponding changes in the control point of a temperature controller whose temperature-sensitive bulb is located in the supply water to the heating panel.

The temperature controller repositions the three-way valve, which mixes hot boiler water with cooler return water from the bypass to provide supply water at the proper temperature as measured by the temperature-sensitive bulb.

The room thermostat, if used, acts as a limit control, responding to changes in load caused by occupancy, solar gain, and the like.

The circulator operates continuously so long as the three-way valve is positioned for any degree of heating.

The boiler water temperature is maintained at the proper level for the indirect heater, by a temperature controller (immersion Aquastat) which operates the burner.

The safety control prevents excessive boiler water temperature.

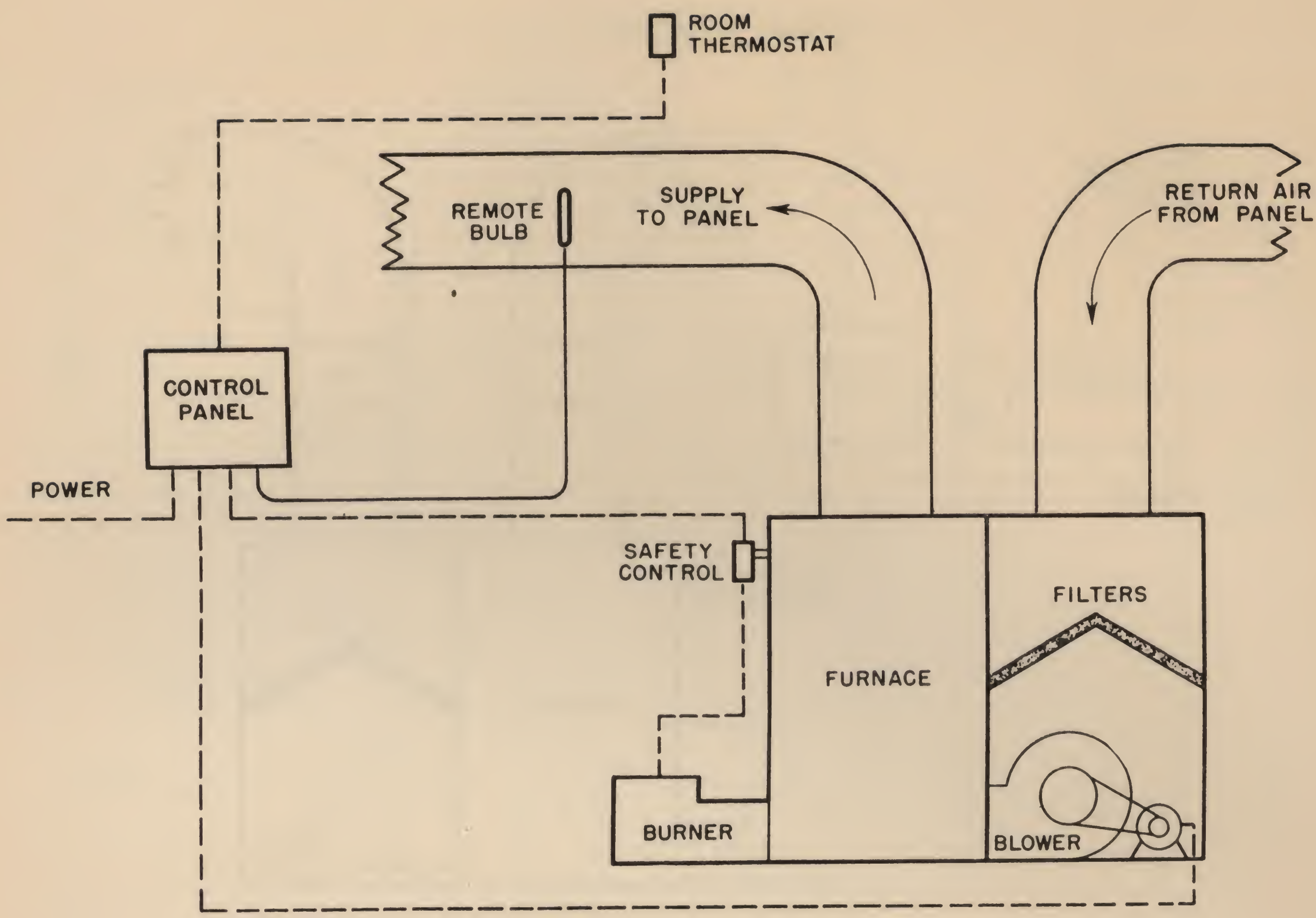


Fig. 10—Single Zone Electric—Forced Warm Air—Light Panel

The room thermostat responds to any changes in heating load by transmitting a signal to the control panel which in turn resets the control point of a temperature controller whose temperature-sensitive bulb is located in the main warm air supply duct to the heating panel.

The temperature controller operates the burner to furnish air to the radiant panel at the temperature called for by the setting of the temperature controller.

The blower operates continuously except when no heat is required.

The safety control will shut off the burner whenever necessary to prevent the temperature in the furnace from becoming excessive.

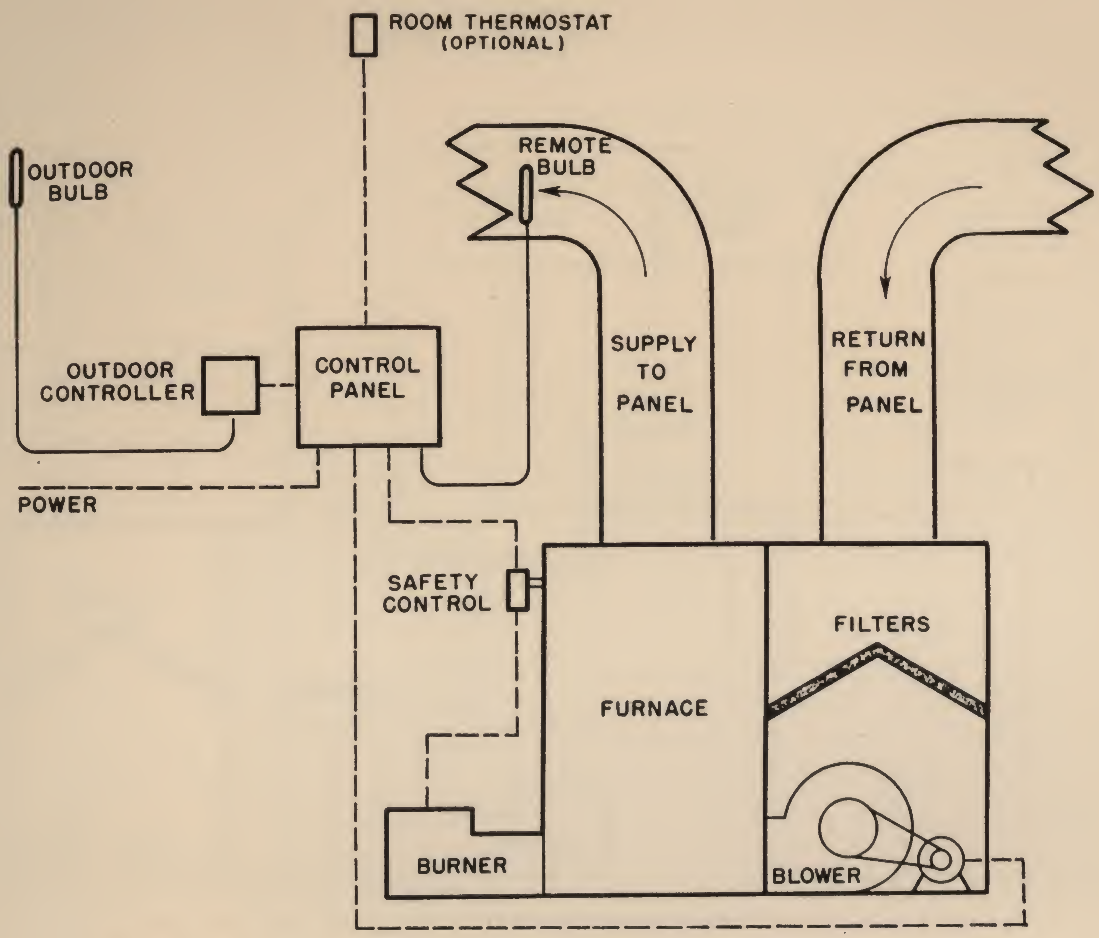


Fig. 11—Single Zone Electric—Forced Warm Air—Heavy Panel

The outdoor controller measures changes in heating load as represented by outdoor temperatures and transmits them to the control panel, which makes commensurate adjustments in the control point of a temperature controller whose temperature-sensitive bulb is located in the main supply duct to the radiant panel.

The temperature controller operates the burner to furnish air to the panel at the proper temperature as measured by the sensitive bulb.

The room thermostat (if used) measures changes in load caused by occupancy, solar gain, or the like, and acts as a limit control on room temperatures.

The blower is shut down by an auxiliary switch in the control panel whenever no heat is required.

The safety control acts to prevent excessive furnace temperatures.

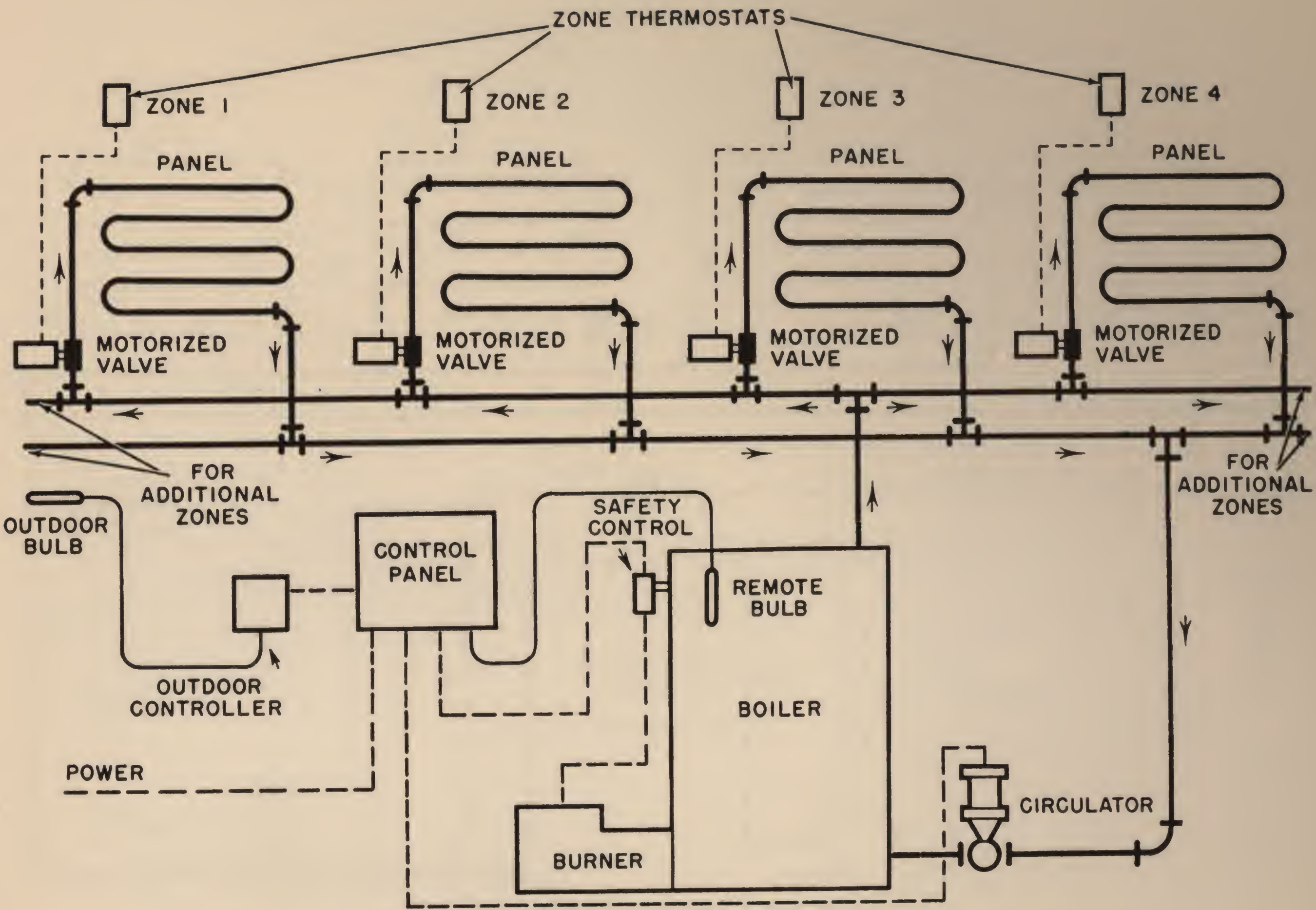


Fig. 12—Multiple Zone Electric—Forced Hot Water—Without Indirect Heater

The outdoor controller responds to a change in heating load as determined by the outdoor temperature, by transmitting a signal to the control panel. A corresponding change is made in the control point setting of a temperature controller whose sensitive bulb is located in the boiler water.

The temperature controller operates the burner to furnish boiler water of the proper temperature as measured by the sensitive bulb.

Each zone thermostat controls a motorized valve in the panel supply line for the zone, so as to regulate the panel temperature in accordance with the individual heating requirements of the zone. Modulating zone valves are generally preferred, but two-position valves may be used when the characteristics of the installation so require.

The circulator runs continuously except when no zone requires heat.

The safety control will shut off the burner whenever for any reason the boiler water reaches the maximum safe or desirable temperature.

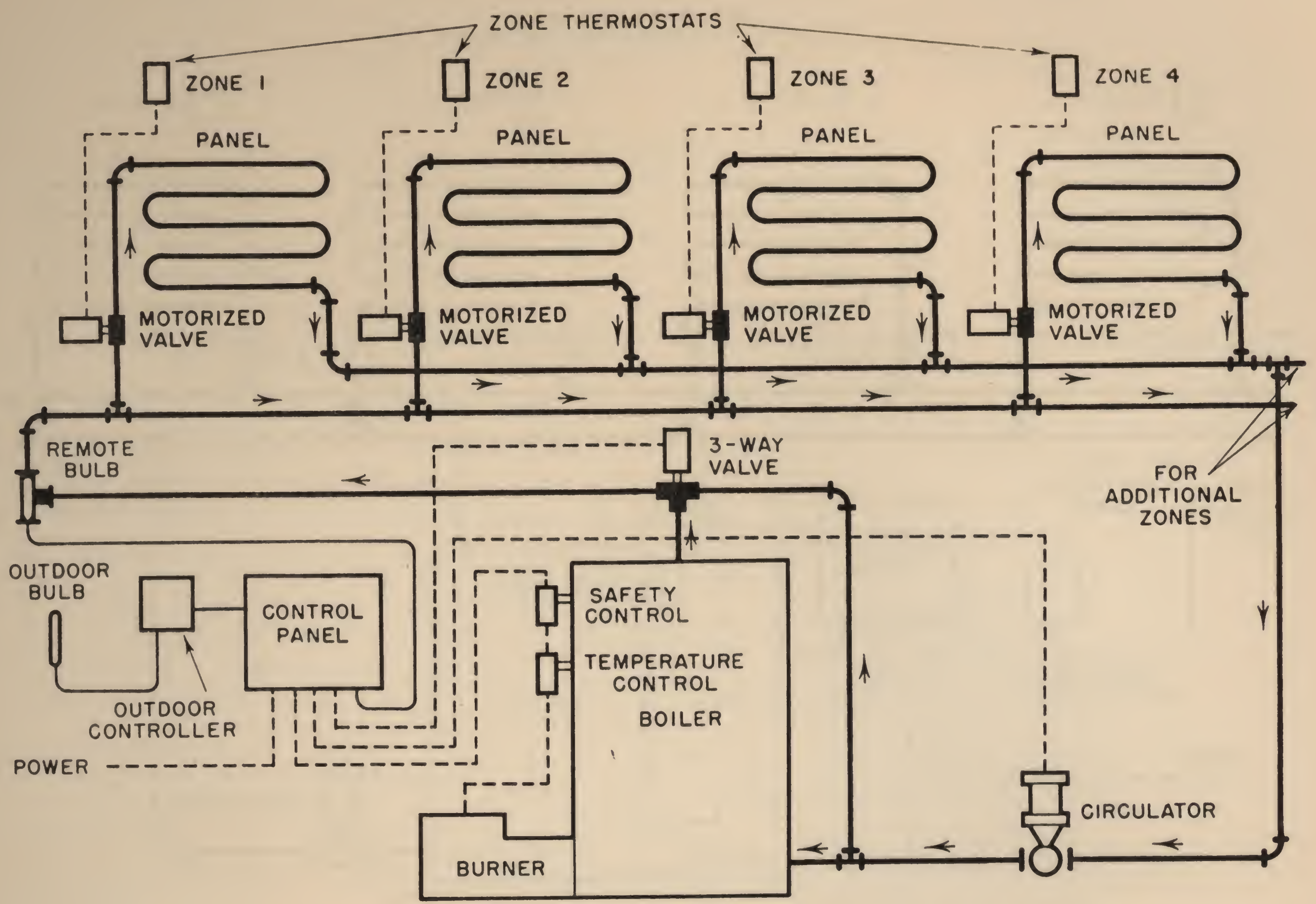


Fig. 13—Multiple Zone Electric—Forced Hot Water—With Indirect Heater

The outdoor controller measures changes in heating load as represented by outdoor temperatures. These changes, transmitted to the control panel, cause corresponding changes in the setting of a temperature controller whose temperature-sensitive bulb is located in the main line to the distribution system, downstream from the three-way valve.

The temperature controller positions the three-way valve, which mixes hot boiler water and cooler return water in the proportions required by the setting of the temperature controller, so as to furnish supply water at a temperature commensurate with the heating load.

A temperature controller located in the boiler operates the burner so as to maintain the boiler water temperature required by the indirect heater.

A thermostat in each zone measures the individual heating requirements of the zone and controls a motorized valve—modulating or two-position as the installation may require—in the zone supply line; the zone valve thus regulates the panel temperature in accordance with the individual heating requirements of the zone.

The circulator operates continuously so long as any heating is required.

The safety control will shut off the burner whenever necessary to prevent excessive temperatures in the boiler.

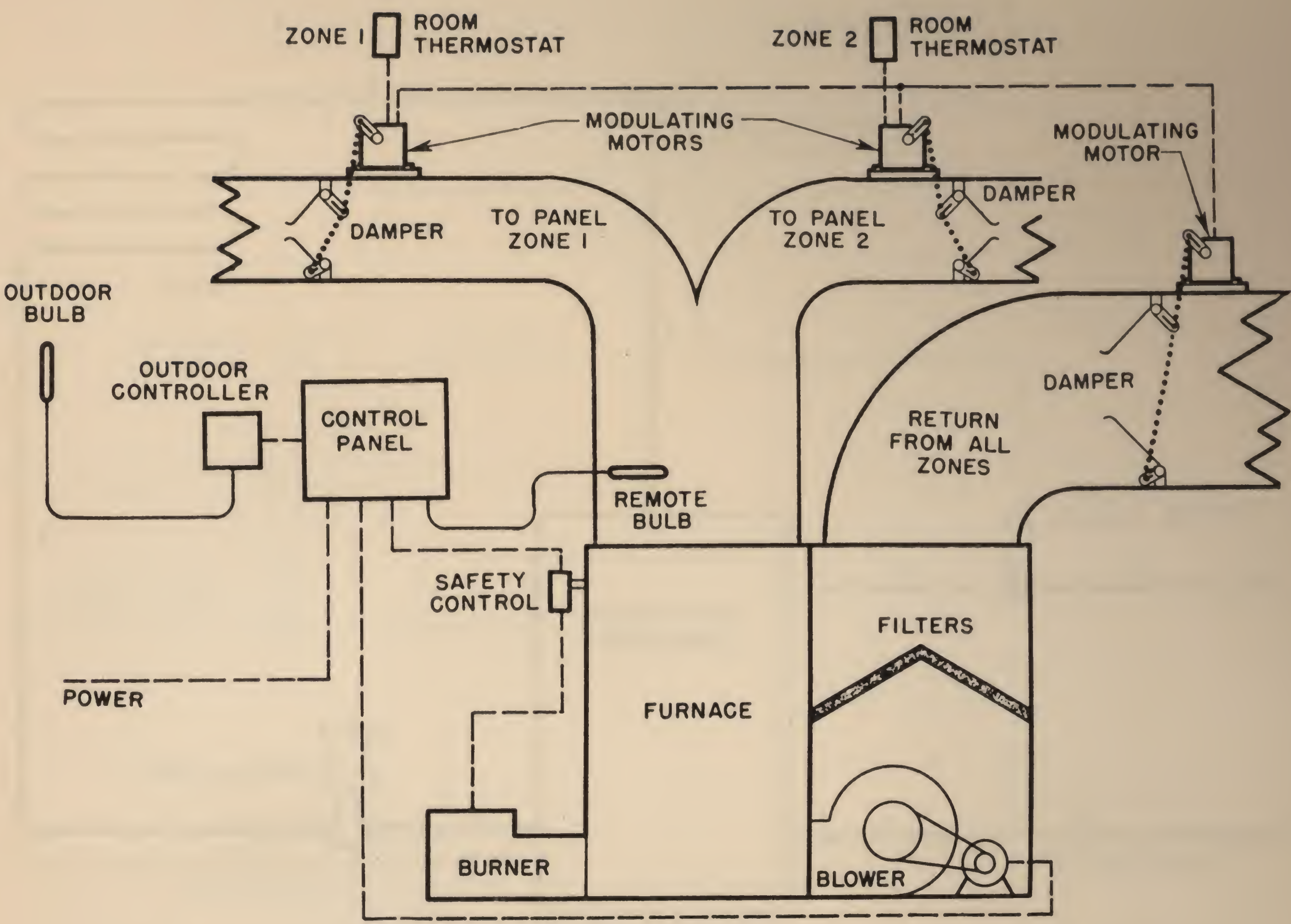


Fig. 14—Multiple Zone Electric—Forced Warm Air—Light or Heavy Panel

The outdoor controller transmits to the control panel changes in the heating load as determined by outdoor temperatures. The control panel resets the control point of a temperature controller whose sensitive bulb is located in the main discharge duct or plenum chamber.

The temperature controller operates the burner to supply air to the system at the temperature called for by the setting of the temperature controller.

A thermostat in each zone measures changes in the individual heat load in that zone, and makes corresponding changes in the position of the modulating damper motor so as to regulate the volume of warm air delivered to the zone panel. A modulating motor linked to a volume damper in the main return duct is positioned according to the total demand of all the zone dampers, so as to maintain a constant pressure balance in the system.

The blower operates continuously except when no heating is required.

The safety control acts to prevent excessive furnace temperatures.

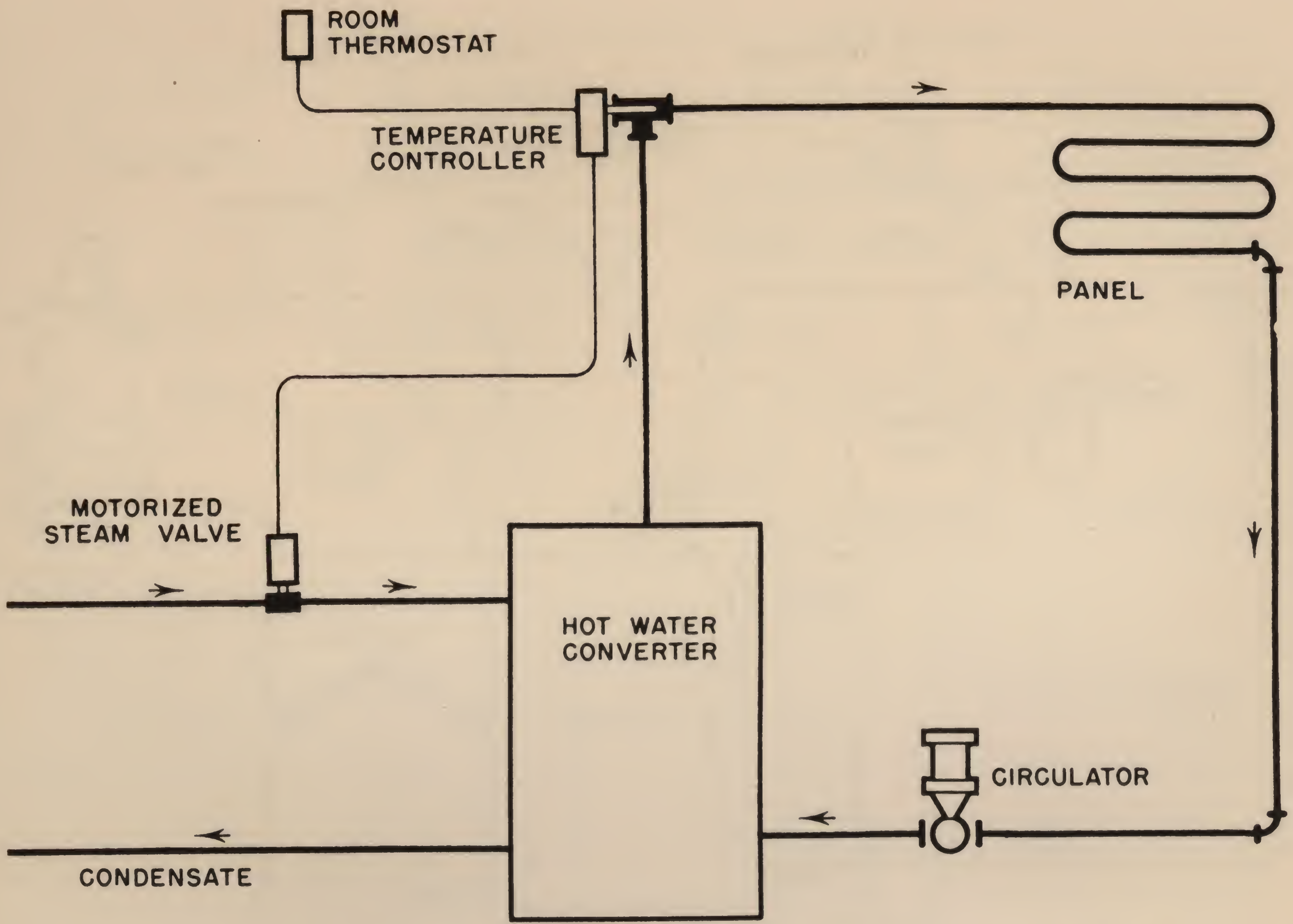


Fig. 15—Single Zone Pneumatic—Forced Hot Water—With Converter—Light Panel

Changes in the heating load as measured by the room thermostat are transmitted to the temperature controller whose temperature-sensitive bulb is located in the main supply line to the panel (or in the converter).

The temperature controller actuates the motorized valve in the steam supply line to the converter, so as to provide a supply water temperature commensurate with the heating load.

The circulator operates constantly as long as heat is required.

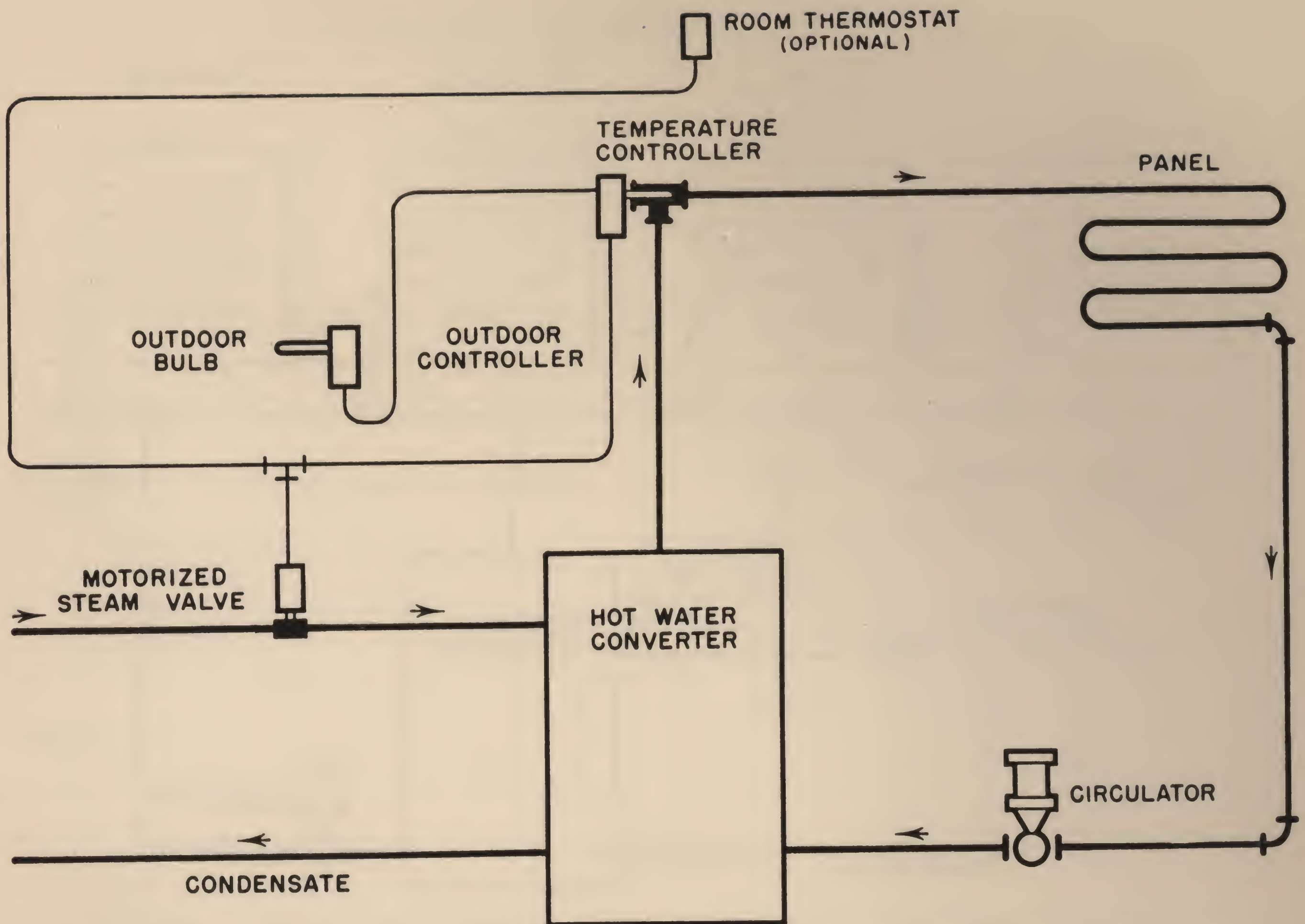


Fig. 16—Single Zone Pneumatic—Forced Hot Water—With Converter—Heavy Panel

The outdoor controller measures changes in the heating load as determined by outdoor temperatures and transmits these changes to the temperature controller whose bulb is located in the main supply line to the heating panel (or in the converter).

The temperature controller in turn actuates a motorized valve in the steam line to the converter so as to maintain a supply water temperature corresponding to the heating load.

The room thermostat, if used, measures variations in heating load as a result of occupancy, solar gain, etc., and functions as a limit control.

The circulator operates continuously except when no heat is required.

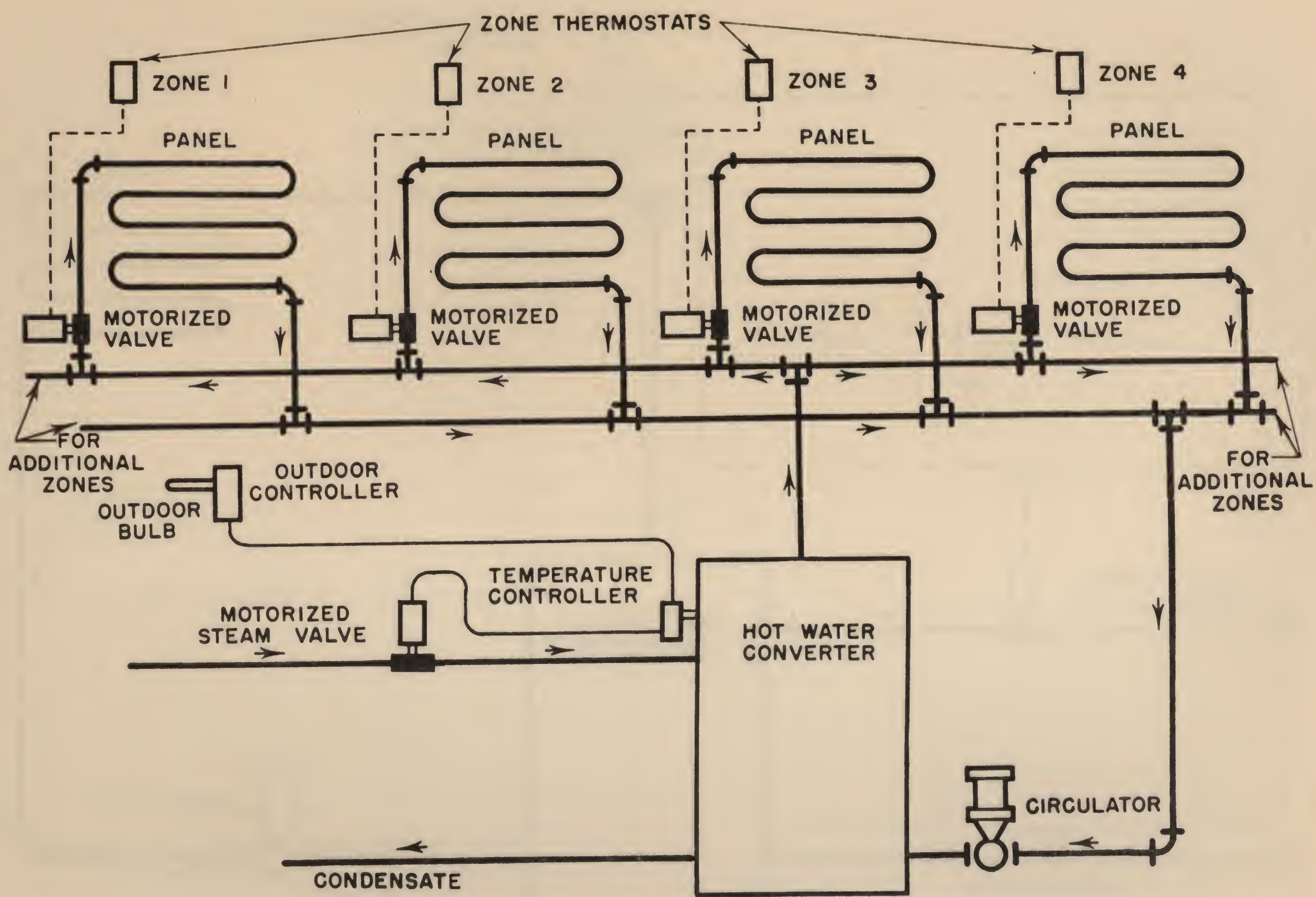


Fig. 17—Multiple Zone Pneumatic—Forced Hot Water—Variable Converter Temperature

The outdoor controller measures changes in heating load as determined by outdoor temperatures, and makes corresponding changes in the control point of the temperature controller whose bulb is located in the hot water converter.

The temperature controller in turn actuates the motorized steam valve to provide just sufficient steam to the converter for maintaining the water temperature called for by the temperature controller setting.

A thermostat in each zone measures variations in zone heating requirements and controls a motorized valve in the supply line to the zone. The zone valve, which may be of the modulating or the two-position type according to the characteristics of the installation, adjusts the supply of water to the zone according to the panel temperature required by the individual zone heating load.

The circulator operates continuously except when no heat is required.

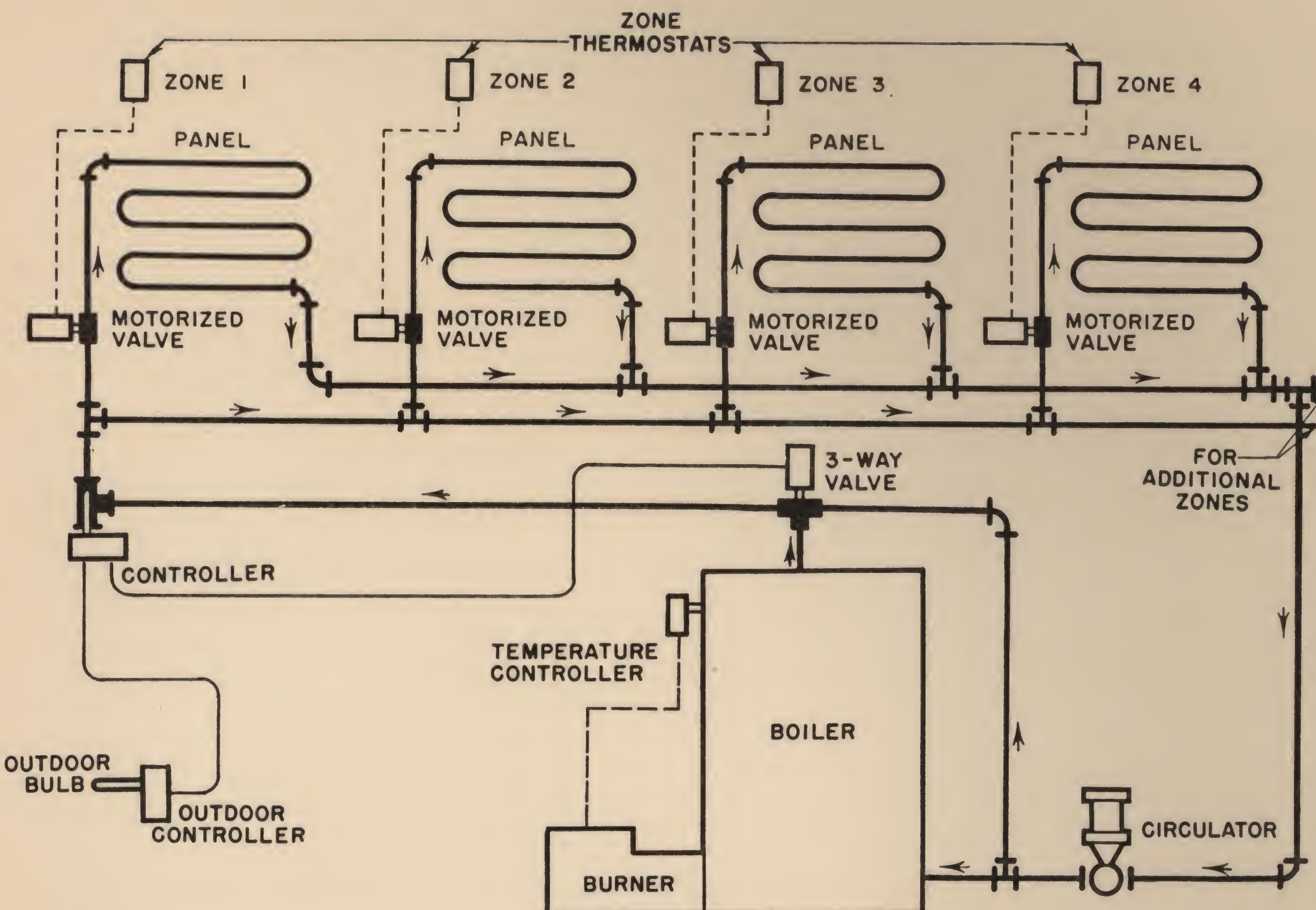


Fig. 18—Multiple Zone Pneumatic—Forced Hot Water—Fixed Boiler Temperature

The system shown is applicable where the boiler (or converter) supplies hot water to some other equipment—such as unit heaters or heating coils in mechanical ventilation systems—in addition to the radiant panels. A temperature controller whose bulb is located in the boiler (or hot water converter) operates the burner (or steam valve) to maintain the proper water temperature.

The outdoor controller measures any change in the heating load as determined by the outdoor temperature and resets a temperature controller whose bulb is located in the discharge line of the three-way valve.

This temperature controller accordingly repositions the three-way valve, which mixes hot water and return water in the proportions required by the heating load as reflected in the control point of the discharge temperature controller.

The zone thermostats operate the zone motorized valves—modulating or two-position as the installation may require—so as adjust the supply to the individual zones and consequently the panel temperatures in accordance with the variations in zone heating requirements.

The circulator operates continuously except when no heat is required.